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Summary of Ph.D. Thesis

Analysis and Synthesis of Time Delay System Spectrum

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ANOTACE

Použití lineárních modelů pro popis reálných systémů je konvenčním přístupem používaným v oboru automatického řízení. Pomocí klasického lineárního modelu můžeme popsat pouze část dynamiky systému (dynamiku v okolí tzv. pracovního bodu). Výhoda lineárních modelů spočívá v jejich snadné manipulovatelnosti a snadném návrhu řízení pomocí metod dostupných pro tuto třídu modelů. Hlavní nevýhoda těchto modelů, kromě jejich omezené platnosti, je dána tím, že jejich jedinými dynamickými elementy jsou integrátory aproximující soustředěné akumulace. Z toho vyplývá že popis systémů s rozloženými parametry nebo s fenoménem dopravního zpoždění (se kterými se setkáváme v technické praxi) je pomocí tohoto přístupu poněkud problematický. Mnohem lepších výsledků při modelování takovýchto systémů lze dosáhnout zahrnutím dopravních zpoždění do struktury lineárního modelu. Takto získaný popis nabízí větší variabilitu struktury modelu než klasický popis, což umožňuje přesněji popsat dynamiku systému. Na druhou stranu, analýza dynamiky modelů s dopravním zpožděním a syntéza řízení je zpravidla složitější než u klasického přístupu. Typickou vlastností modelů s dopravním zpožděním je nekonečné spektrum vlastních hodnot (pólů a nul) systému. V této práci je navržena metoda analýzy dynamiky systémů s dopravním zpožděním na základě znalosti spekter pólů a nul systému. Pro výpočet pólů a nul je navržen původní algoritmus založený na mapování charakteristických funkcí systému. Důležitost jednotlivý pólů je posuzována na základě váhových funkcí přenosů prvního a druhého řádu které jsou získány rozložením přenosu systému použitím Heavisideovy věty o rozkladu. Tímto způsobem je možné definovat skupinu pólů které hrají v dynamice systému rozhodující roli. Určení dominantních pólů systému umožňuje nejenom analyzovat módy systému, ale také dynamiku systému výhodně změnit přesunutím těchto dominantních pólů. I u systémů s dopravním zpožděním je možné dynamiku změnit zavedením zpětných vazeb od stavových proměnných systému. Na druhou stranu je nutné podotknout že touto metodou můžeme umístit pouze malou část spektra pólů (uvažujeme-li koeficientové zpětné vazby). V této práci je proveden rozbor této metody řízení při aplikaci na systémy s dopravním zpožděním a je předložena metoda efektivního návrhu zpětných vazeb. Snadná aplikovatelnost metody pro analýzu dynamiky systému s dopravním zpožděním a efektivnost návrhu koeficientů zpětných vazeb od stavových veličin přesunutím dominantních pólů jsou ukázány v aplikačním příkladu kde analyzovaným systémem je laboratorní tepelná soustava.

ABSTRACT

The classical approach used in modelling of the real plant dynamics in the field of control engineering is based on the linear model. Since the dynamics of the real plants are non-linear as a rule, the linear model fits the dynamics of the plant only in a vicinity of the operational point at which the system has been identified. The linear model is easy to handle and the control design is easy to perform using a method available for this class of systems (models). Its main drawback is given by the fact that the only dynamical elements of the model are the integrators representing the point accumulations. Thus, using this modeling approach, it is difficult to fit the dynamics of plants with the distributed parameters or with the transportation phenomenon involved. Much better results in modeling of this class of systems are achieved by involving the time delays in the structure of the model. Such a model with more variable structure (called time delay system) provides the opportunity to fit better the plant dynamics than delay free linear model. On the other hand, the analysis of the dynamics of time delay system and the control synthesis is more complicated as a rule. The typical features of time delay systems are the infinite spectra of poles and zeros. In this thesis, the methodology for analyzing the dynamics of time delay system is introduced based on the knowledge of the significant parts of the spectra of poles and zeros. An original algorithm for computing poles and zeros of the system with delays is designed based on the mapping the characteristic functions of the system. The significance of the poles is evaluated on the basis of the weighting functions corresponding to the first and second order transfer functions resulting from applying the generalized Heaviside expansion to the transfer function of time delay system. In this way, it is possible to define a group of the dynamics determining poles. Assessing the group of the most significant poles allows not only the modes to be analyzed but also the dynamics to be positively changed by shifting these poles into more favorable positions. In the same way as in the case of classical delay free systems, this shifting of the poles can be accomplished using the coefficient feedback loops from the state variables. However, using this pole placement method, only few poles can be prescribed while the rest of infinitely many poles are placed spontaneously. In this thesis, the features of the pole placement method using the coefficient feedback loops from the state variables applied to time delay systems are investigated and an effective method for feedback design is presented. In order to demonstrate that the method for analyzing the dynamics of systems with time delays and the extension of pole placement method described in this thesis are easily applicable, an application example, in which the system being analyzed is a laboratory heating system, is included in the end of this thesis.

LIST OF SYMBOLS

$\mathcal{A}$	infinitesimal generator of the semigroup of autonomous TDS
$\mathbf{A}(\tau)$	functional matrix of system dynamics
$\mathbf{A}(s)$	Laplace transform of $\mathbf{A}(\tau)$
$\mathcal{A}_h$	discrete approximation of $\mathcal{A}$ for h
$\mathbf{B}(\tau)$	system input functional matrix
$\mathbf{B}(s)$	Laplace transform of $\mathbf{B}(\tau)$
$\mathcal{C}$	Banach space
$\mathbb{C}$	complex space
$\mathbf{C}$	matrix of system outputs
$\mathcal{D}$	suspect region of the complex plane in which the function roots are computed
$D(s)$	exponential polynomial, Laplace transform of the difference function
$G(s)$	transfer function of the model approximated real plant dynamics
h	sampling period, step of the discretization
$\mathbf{H}_i$	matrices of the difference equation associated with the neutral system
$H_i(s)$	transfer functions of the system modes resulting from the Heaviside expansion
$h_i(t)$	weighting functions of $H_i(s)$
$h_{\mathrm{e}i}$	pole-significance evaluating criterion based on evaluating the weighting functions of the transfer functions corresponding to the modes of TDS
$\mathbf{I}$	identity matrix
$I(\beta,\omega)$	imaginary part of $M(\beta+\mathrm{j}\omega)$ ($N(\beta+\mathrm{j}\omega)$, $D(\beta+\mathrm{j}\omega)$)
$\mathrm{Im}(\cdot)$	imag part of $\cdot$
$\mathbf{K}(s)$, $\mathbf{K}$	feedback matrix
$M(s)$	denominator of $G(s)$, characteristic function of the system (quasi)polynomial
$N(s)$	numerator of $G(s)$, (quasi)polynomial
$\mathbb{R}$	real space
$R(\beta,\omega)$	real part of $M(\beta+\mathrm{j}\omega)$ ($N(\beta+\mathrm{j}\omega)$, $D(\beta+\mathrm{j}\omega)$)
$R(\lambda_i)$	residues corresponding to λ_i
$\mathrm{Re}(\cdot)$	real part of $\cdot$
s	complex variable, operator of Laplace transform
s_i	root of the function in with argument s
$\mathcal{T}(t)$	solution operator of autonomous TDS for t
t	time
$\mathbf{u}(t)$	vector of system input variables
$\mathbf{u}(s)$	Laplace transform of $\mathbf{u}(t)$
$\mathbf{x}(t)$	vector of system state variables
$\mathbf{x}(s)$	Laplace transform of $\mathbf{x}(t)$
$\boldsymbol{x}_t$	state of TDS at time t
$\boldsymbol{x}_k$	state of the discrete system corresponding to TDS at discrete time k
$\mathbf{y}(t)$	vector of system output variables
$\mathbf{y}(s)$	Laplace transform of $\mathbf{y}(t)$
z	operator of Z transform, complex variable
z_i	poles of discretized system
$\Delta(\lambda)$	characteristic matrix of the system
$\Phi(s)$	argument variable
$\mathbf{\Phi}$	matrix of discrete approximation of $\mathcal{T}(h)$
υ_i	eigenvalues of the essential spectrum of neutral system
λ	complex variable
λ_i	poles of the system
σ_i	prescribed spectrum of poles
μ_i	zeros of the system
TDS	time delay system(s)

CONTENTS

1. INTRODUCTION, THE ACTUAL STATE OF RESEARCH AND THE THEORETICAL BACKGROUND OF TIME DELAY SYSTEMS

Time delay systems (TDS) provide an alternative way for building the models of real plants to the classical system description given by a set of ordinary differential equations. Besides the integrators, involving the delays as the other dynamical elements brings about favourable features in fitting real plant dynamics. On the other hand, the functionality of the system matrices (considering the linear models) results in infinite spectra of the system poles and zeros. This inconvenient feature of time delay systems cause difficulties in the analysis and control design of TDS.

First of all, let us briefly outline some of the literature sources of the theory of TDS. Already in Myshkis, (1949) the theory of a general class of differential equations with delayed arguments has been introduced. From the further books, the monographs of Krasovski (1963), Bellman and Cooke (1963), El'sgol'c and Norkin (1971), Hale (1977) are the fundamental sources of the knowledge in the field of time delay systems. From the more recent monographs dealing with this subject, let us mention Kolmanovski and Nosov (1986) (stability, application examples), Górecki, et al, (1989) (analysis and synthesis) and Diekman, et al., (1995) (operator theory approach). The recent comprehensive introductions are (Kolmanovski and Myshkis, 1992) or (Hale and Verduym Lunel, 1993).

Time delay systems belong to the class of infinite dimensional systems (Bensoussan, et al, 1993). The modelling approach using time delays is largely used to describe propagation and transport phenomena, which can be met in the applications throughout the fields of mechanical, chemical and electrical engineering. Other typical areas of the application of time delay systems are populations dynamics and the economics. There are several approaches to describe TDS as the infinite dimensional system. Regarding the practical point of view, in this thesis, the description based on linear functional differential equations is used (Górecki, et al., 1989), (Hale and Verduym Lunel, 1993). Using the Stieltjes integrals, a general form of the TDS description is considered in the form

$$\frac{d\mathbf{x}(t)}{dt} = \sum_{i=1}^{N}\left[\mathbf{H}_i \frac{d\mathbf{x}(t-\eta_i)}{dt}\right] + \int_0^T d\mathbf{A}(\tau)\mathbf{x}(t-\tau) + \int_0^T d\mathbf{B}(\tau)\mathbf{u}(t-\tau),\ \ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{1}$$

where $\mathbf{x}\in \mathrm{R}^n$ is the vector of state variables, $\mathbf{u}\in \mathrm{R}^m$ is the vector of system inputs, $\mathbf{y}\in \mathrm{R}^p$ is the vector of system outputs, $\mathbf{A}(\tau)$, $\mathbf{B}(\tau)$, $\mathbf{H}_i$, $\mathbf{C}$ are the matrices of compatible dimensions, $\eta_1 < \eta_2 < ... < \eta_N < T$ and τ is delay variable. The distribution functions of the delays are involved in the functional matrices $\mathbf{A}(\tau)$, $\mathbf{B}(\tau)$. Description of TDS based on the use of the Stieltjes integrals is the basic modelling approach known as anisochronic approach, see Zítek, (1983, 1997, 1998). Functional description (1) acquires more convenient form after applying the Laplace transform (considering zero initial conditions)

$$s\mathbf{x}(s) = \left[s\sum_{i=1}^{N}\left[\mathbf{H}_i \exp(-s\eta_i)\right] + \mathbf{A}(s)\right]\mathbf{x}(s) + \mathbf{B}(s)\mathbf{u}(s) \tag{2}$$

where $\mathbf{A}(s) = \int_0^T \exp(-s\tau)d\mathbf{A}(\tau),\ \mathbf{B}(s) = \int_0^T \exp(-s\tau)d\mathbf{B}(\tau)$

Since also the derivations of the state variables are delayed in (1), the model is in the form of neutral system. If $\mathbf{H}_i = \mathbf{0}$, $i = 1,2,..,N$, model (1) is called retarded system. Note that the vector of state variables does not represent state of TDS (1). In fact, the state of system (1) is given by the function segments of the system state variables and the system inputs $\{\boldsymbol{x}_t, \boldsymbol{u}_t\}$ on the segment of the last system history, see, e.g., Górecki, et al., (1989), Zítek, (1998) where

$$\boldsymbol{x}_t(\tau) = \mathbf{x}(t+\tau),\ \boldsymbol{u}_t(\tau) = \mathbf{u}(t+\tau)\quad -T \le \tau \le 0 \tag{3}$$

and the state space is the Banach space of continuous real functions on the interval of length T, $\mathcal{C} = C([-T,\ 0],\mathrm{R}^n)$, see Hale and Verduyn Lunel, (1993). If the subject of the analysis is the investigation of the homogeneous system dynamics, e.g., distribution of the system poles, it is convenient to analyse the dynamics of autonomous system (the system inputs are omitted). It allows the abstract operator theory to be used in the analysis, e.g., the semigroup approach, see, Diekman, et al., (1995), Hale and Verduym Lunel, (1993). Let us consider the autonomous retarded system

$$\frac{d\mathbf{x}(t)}{dt} = \int_0^T d\mathbf{A}(\tau)\mathbf{x}(t-\tau), \quad \mathbf{x}(t) = \varphi(t) \quad t \in [-T,\ 0] \tag{4}$$

The state of autonomous system (4) given by $\boldsymbol{x}_t$ is uniquely determined by the initial condition function φ, i.e. state at $t = 0$, by the solution operator, thus

$$\mathcal{T}(t)\varphi = \boldsymbol{x}_t, \quad \varphi \in \mathcal{C} \tag{5}$$

Operator $\mathcal{T}(t)$, maps the initial state φ at time zero to $\boldsymbol{x}_t$. Family $\mathcal{T}$ is called the strongly continuous semigroup, which is given by the translation along the solutions of (4). Description (5) can be rewritten into

$$\frac{d}{dt}\boldsymbol{x}_t = \mathcal{A}\boldsymbol{x}_t,\ t > 0,\ \boldsymbol{x}_0 = \varphi \tag{6}$$

where $\mathcal{A}$ is generator of the semigroup. The mutual relationship between $\mathcal{T}(t)$ and $\mathcal{A}$ is given by

$$\mathcal{A}\phi = \lim_{t\to 0}\frac{1}{t}\left(\mathcal{T}(t)\phi - \phi\right) \tag{7}$$

As will be shown later, discretizing either $\mathcal{T}(t)$ or $\mathcal{A}$ is one possible way to compute the approximations of the poles of retarded systems. The notion of poles and zeros of TDS is identical as in the case of delay free models. The only difference is that due to the functionality of the matrix $\mathbf{A}(s)$ and $\mathbf{H}(s) = \sum_{i=1}^{N}\mathbf{H}_i \exp(-\eta_i s)$, the spectrum of poles is infinite. Also the spectrum of zeros may be infinite, depending on the model structure. Let system (2) is transformed into the form of transfer matrix

$$\mathbf{G}(s) = \frac{\mathbf{y}(s)}{\mathbf{u}(s)} = \mathbf{C}\left[s(\mathbf{I} - \sum_{i=1}^{N}[\mathbf{H}_i \exp(-s\eta_k)]) - \mathbf{A}(s)\right]^{-1}\mathbf{B}(s) \tag{8}$$

The system poles, which are common for all the transfer functions in $\mathbf{G}(s)$, are given as the solutions of

$$M(s) = (\det s(\mathbf{I} - \sum_{i=1}^{N}[\mathbf{H}_i \exp(-s\eta_k)]) - \mathbf{A}(s)) = 0 \tag{9}$$

on the other hand, each of $G_{k,l} = N_{kl}(s)/M(s)$ has its own set of zeros given as the solutions of

$$N_{kl}(s) = \mathbf{C}_k \operatorname{adj}\left[s(\mathbf{I} - \sum_{i=1}^{N}[\mathbf{H}_i \exp(-s\eta_k)] - \mathbf{A}(s)\right]\mathbf{B}_l(s) = 0,\ k = 1..p,\ l = 1..m \tag{10}$$

where $\mathbf{C}_k$ is the row sub-matrix of $\mathbf{C}$ corresponding to the k^{th} output and $\mathbf{B}_l(s)$ is the column sub-matrix (vector) corresponding to the l^{th} input. Note that both $M(s)$ and $N(s)$ are quasipolynomials.

The features of the spectrum of the poles depend on the character of the system. If the system is retarded, the number of poles with real parts greater than $\alpha \in \mathrm{R}$ is always finite. This convenient feature implies that number of unstable poles (the poles located to the right from the stability boundary) is always finite. The poles are distributed in asymptotic chains with the following features. Let all the poles of system (8) be ordered in a sequence $\lambda_1, \lambda_2, \ldots, \lambda_k$ with respect to their magnitudes, then $|\lambda_k| \to \infty$ and $\mathrm{Re}(\lambda_k) \to -\infty$ as $k \to \infty$. If the system is neutral, the pole spectrum has very different features. First, the distribution of poles is determined by the essential spectrum of the system. The essential spectrum, i.e., the eigenvalue spectrum of the difference equation associated with the neutral system, is given as the solutions of the following equation

$$D(s) = \det\left[\mathbf{I} - \sum_{k=1}^{N}\mathbf{H}_k \exp(-s\eta_k)\right] = 0 \tag{11}$$

The features of the root spectrum υ_k of the exponential polynomial $D(s)$ have been investigated, e. g., by Avelar and Hale, (1980). First, there exist $\alpha \in \mathrm{R}$ and $\beta \in \mathrm{R}$ such that $\alpha < \mathrm{Re}(\upsilon_k) < \beta,\ \ k = 1, 2, \ldots, \infty$, i.e., the roots of (11) are distributed in a vertical strip. Also in the pole spectrum of a neutral system there exists a sequence of poles λ_k such that $|\lambda_k| \to \infty$ as $k \to \infty$. There also exists a sequence υ_k of roots of (11) such that $(\lambda_k - \upsilon_k) \to 0$ as $k \to \infty$. Thus, if there are υ_k located on the right half of the complex plane, the neutral is not only unstable, but it is unstable with infinitely many poles. If the system is retarded, the system poles, i.e., the roots of the characteristic equation are the eigenvalues of the matrix $\mathbf{A}(s)$ and of the generator $\mathcal{A}$. The poles of retarded system are also related to the eigenvalues z_i of the solution operator $\mathcal{T}(t)$ by the relation

$$z_i = \exp(\lambda_i t)\,,\ i = 1,2,... \tag{12}$$

Since as well $\mathcal{A}$ as $\mathcal{T}(t)$ may be discretized using a numerical method, which provides finite order discrete system, the approximation of the rightmost poles can be computed as the eigenvalues of the discretized system. First, let us consider the discrete state of the system given as the samples of $\mathbf{x}(t)$ on the last system history with the time spacing determined by the sampling period h

$$\boldsymbol{x}_k = \left[\mathbf{x}_{k,}\ \mathbf{x}_{k-1},....,\mathbf{x}_{k-H+1},\mathbf{x}_{k-H}\right]^T \tag{13}$$

where k is the discrete time and H is determined by the maximum delay of the system and the order of the numerical method used for the discretization. The system state at k+1 is given by

$$\boldsymbol{x}_{k+1} = \boldsymbol{\Phi}\boldsymbol{x}_k \tag{14}$$

which is the discrete form of

$$\boldsymbol{x}_{t+h} = \mathcal{T}(h)\boldsymbol{x}_t \tag{15}$$

where $\boldsymbol{\Phi} \in \mathrm{R}^{(H+1)n\times(H+1)n}$ is the discrete approximation of $\mathcal{T}(t)$. Therefore, the approximations of the rightmost poles of retarded system are obtained as the rightmost eigenvalues of $\boldsymbol{\Phi}$ transformed into s-plane on the basis of (12). Analogously, also discrete form of (6) can be obtained as

$$\frac{d}{dt}\boldsymbol{x}_k = \mathcal{A}_h \boldsymbol{x}_k \tag{16}$$

where $\mathcal{A}_h$ is the discretized form of $\mathcal{A}$. The method of computing the approximate values of the rightmost poles based on discretization of the solution operator using linear multi-step methods has been worked out by Engelborghs and Roose (1999), Engelborghs, et al, (2000), see also Engelborghs and Roose (2002) and the manual of the DDE-Biftool, (Engelborghs, et al, 2001, the Matlab package for bifurcation analysis of delay differential equations). The other approach, i.e., computing the approximations by means of discretizing the generator $\mathcal{A}$ has been solved by Ford and Wulf, (1988), Wulf and Ford, (2000), see also Breda, et al., (2001).

On the basis of the spectrum of poles of TDS, we can investigate the system dynamics through the analysis of the modes of the system. The motivation is to define the most significant poles of the system (even though the system has infinitely many poles, only few of them are decisive in the system dynamics) and decide whether or not their positions imply favourable features of the modes of the dynamics. If it is not so, the distribution of the poles, i.e., the system dynamics, may be changed by introducing the feedback loops from the state variables

$$u = -\mathbf{K}\mathbf{x}(t) \tag{17}$$

The features of the extension of classical state feedback design by the gain coefficient feedback to the class of TDS has been studied by Zítek, (1997, 1998), Zítek and Vyhlídal, (2000, 2002). The idea of applying such a pole assignment consists in selecting system poles $\sigma_i, i = 1..n$ to be assigned in order to stabilize or improve the dynamics of TDS. The gain coefficients K_1, K_2, ... , K_n are then computed from the set of equations

$$M(\sigma_i, K_1, K_2, ..., K_n) = 0,\ i = 1..n \tag{18}$$

where $M(s,\mathbf{K}) = \det(s\mathbf{I} - \mathbf{A}(s) + \mathbf{B}(s)\mathbf{K})$. Since only n poles can be assigned, while the other poles are placed spontaneously, the resultant spectrum must always be investigated before applying the computed gains. If any of the non-prescribed poles become dominant with unfavourable positions, the prescription procedure has to be repeated. Even though the method is rather heuristic, it has proved to be a valuable tool for control design of TDS. An automatic pole placement procedure has been worked out by Michiels, et al., (2002). The method is designed to stabilize the system dynamics by shifting the rightmost poles as far to the left as possible applying small changes in the gain coefficients. If coefficient feedback (17) is not sufficient to stabilize the system dynamics, a functional feedback may be used, see, e.g., Zítek, et al., (2001) which allows the spectrum of poles to be modified more freely. On the other hand, from the application point of view, the functional feedback is not so convenient (easy to implement) as gain feedback (17).

2. OBJECTIVES OF THE THESIS

The main topics of this thesis are the frequency domain based analysis and pole placement control design of time delay systems. Particularly, the stress will be laid on the methods of synthesis and control design based on the knowledge of the dominant part of the spectrum of the system poles and zeros. As has been mentioned in chapter 1, the applicability of the available algorithms is often restricted on a narrow class of systems. For example, the method for computing the approximations of the rightmost poles using either the discretization of the infinitesimal generator of the semigroup or the discretization of the solution operator can be applied only to retarded systems, preferably with the lumped delays. The distribution of the poles in the complex plane determines the stability and the modes of the dynamics. However, the distribution as such is not decisive in determining the significance of the modes of the dynamics. The significance of the modes is also determined by the distribution of the system zeros. Thus, not only the distribution of the system poles but also the distribution of the system zeros should be known to evaluate completely the system dynamics.

- **Objective 1**. The primary task of this thesis is to develop an algorithm for computing both the poles and zeros of TDS located in a chosen region of the complex plane. The algorithm should provide the possibility to solve the task for large class of systems, both retarded and neutral. Considering that both the system poles and zeros are the solutions of the quasipolynomial equations, the algorithm is to be based on computing the roots of quasipolynomials. Since the quasipolynomials (as well as polynomials) tend to be ill-conditioned as their degree increase, the robustness of the algorithm is to be investigated. The result of the investigation should be the definition of the class of TDS for which the algorithm may be used. This task of the thesis is the most important one. The other tasks are chosen in order to verify the features of the algorithm that will be designed.
- **Objective 2**. The modeling approach involving the delays allows the real plants to be described by considerably lower order models (if the order is considered as the number of differential equations) than if an modeling approach based on the delay free models is used. Thus, the second task to be solved in this thesis is the investigation of the features of the low order models as the basic element units in building the plant model. The investigation should result in the mapping of the distribution of the poles and zeros of the low order anisochronic model. The other result of this part of the thesis should be the choice of the suitable structure of the low order anisochronic model able to fit the dynamics of a wide class of the real plants.
- **Objective 3**. The third task is motivated by the fact that the evaluation of the significance of the poles in the infinite spectra based on evaluating the distances of the poles from the stability boundary, which is mostly used to evaluate the significance of the poles, is sufficient only if the stability of the system is evaluated. If the character of the system input-output dynamics is to be evaluated, the criterion is insufficient. The objective is to define a group of system poles (from the infinite spectrum) that determine the system input-output dynamics. Therefore, I am going to try and find the criterion that will truly evaluate the significance of the system poles.
- **Objective 4**. In section 1.3 I have explained some of the methods for control design of TDS. As the last objective of the thesis, the methods based on the pole placement using the proportional feedback from the state variables are to be investigated. The stress should to be laid on comparing the method based on the direct prescription of the poles with the method known as continuous pole placement. Both the control feedback design methods let also be extended to the class of neutral systems. As the result of this part of the thesis, the suitable strategy for pole placement applied to TDS is to be proposed.

In order to summarize the contributions that will be achieved in this thesis and to demonstrate the applicability of the methods that will be developed, the methods are to be applied for analyzing the dynamics of a real (laboratory) plant application. The control methods based on the continuous and the direct pole placement are to be applied to control the plant.

3. ALGORITHMS FOR COMPUTING QUASIPOLYNOMIAL ROOTS

First, let us briefly mention some of the methods used to compute the roots of polynomials. There exists a great deal of methods to solve this task. The algorithms may be roughly classified either as iterative algorithms (theoretically incomplete), e.g., MP-solve (Bini and Fiorentino, 2000), Eigensolve (Fortune, 2001)), or as theoretically complete, e.g., Weyl's algorithm (Pan, 1997). Most of the former mentioned algorithms are based on computing the eigenvalues of polynomial companion matrix. Since the quasipolynomial companion matrix is functional, using analogous approach to compute the roots of quasipolynomials is not convenient since to compute the approximation of the poles of the functional matrix is a quite difficult task as well. On the other hand Weyl's algorithm based on searching a part of the complex plane can be modified for computing the polynomial roots located in a (suspect) region of the complex plane. The main idea of the algorithm is as follows. On the complex plane, the search for the roots starts with an initial suspect region $\mathfrak{D}=[\beta_{\min},\beta_{\max}]\times[\omega_{\min},\omega_{\max}]$, containing all the polynomial roots. Then the region is partitioned into four congruent subregions. At the centre of each of them, the proximity test is performed (Henrici, 1974), i.e., a distance of the closest root from the centre is estimated. If this distance exceeds the length of the diagonal of the subregion then the subregion does not contain any roots and is discarded. If the result is opposite, the subregion is called suspect and undergoes the same recursive process of partitioning into four subregions. In this way, the algorithm is performed recursively until a desired accuracy of the roots is achieved.

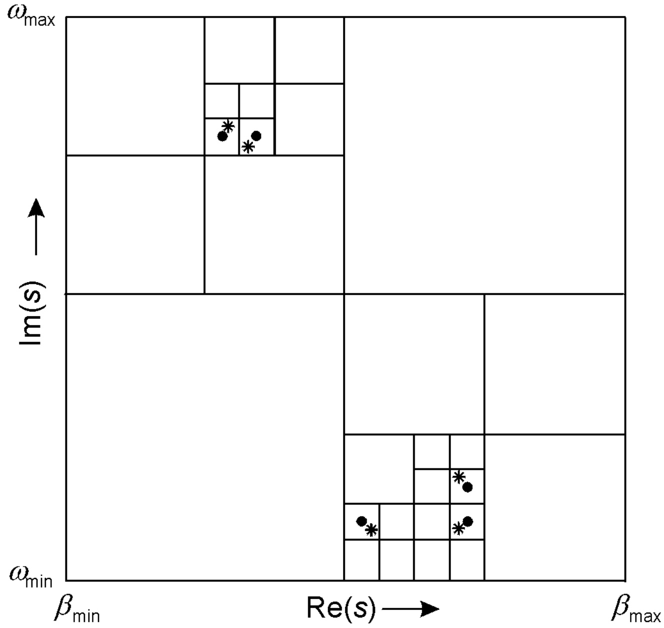

Fig.1 Locating the roots by Weyl's algorithm
asterisks - quasipolynomial roots,
black dots - approximations of the roots

The first algorithm for computing a part of the root spectrum of quasipolynomials I present is a modification of Weyl's construction. The modification consists in using the argument principle based test instead of the proximity test. Argument principle holds for any analytic function including quasipolynomials (El'sgol'ts and Norkin, 1971). Let $\mathfrak{D}$ is a domain in the complex plane whose boundary φ is a closed Jordan curve. Let $G(s)\in\mathrm{C}$ is a meromorphic function, i.e., a single valued analytic function that has no singular points beside poles. Then

$$\frac{1}{2\pi\mathrm{j}}\int_{\varphi}\frac{G'(s)}{G(s)}ds=\frac{1}{2\pi}\Delta_{\varphi}\arg G(s)=N_{\mathfrak{D}}-P_{\mathfrak{D}} \tag{19}$$

where $N_{\mathfrak{D}}$ is the total number of zeros of the function G in $\mathfrak{D}$, $P_{\mathfrak{D}}$ is the total number of poles of the function G in $\mathfrak{D}$, both counting their multiplicities and $G'(s)=dG(s)/ds$.

Consider the quasipolynomial of the form

$$M(s)=\sum_{i=0}^{n}s^{i}Q_{i}(s) \tag{20}$$

where $Q_i(s)$, i=1..n are the functions involving the terms corresponding to the distribution of the system delays. Obviously, quasipolynomial (20) does not have any poles, i.e., $P_{\mathfrak{D}}=0$, thus, the result obtained from applying the argument principle is the number of zeros in the particular region. The algorithm using the Weyl's construction combined with the argument principle based test is quite reliable. On the other hand, if the suspect region is large or there are many roots in the suspect region, the algorithm is rather cumbersome.

The second algorithm I present in the thesis is original, based on the mapping the quasipolynomial function in the complex plane. The quasipolynomial $M(s)$ as a function of the complex variable $s=\beta+\mathrm{j}\omega$ can be split into real and imaginary parts

$$M(\beta,\omega)=R(\beta,\omega)+\mathrm{j}I(\beta,\omega) \tag{21}$$

where $R(\beta,\omega)=\mathrm{Re}\{M(\beta,\omega)\}$ and $I(\beta,\omega)=\mathrm{Im}\{M(\beta,\omega)\}$. Consequently, equation $M(s)$ can be split into

$$R(\beta,\omega)=0,\quad I(\beta,\omega)=0 \tag{22}$$

From the geometric point of view, the roots of $M(s)$ are the intersection points of the curves described by the implicit functions $R(\beta,\omega)=0$ and $I(\beta,\omega)=0$. Mapping the surfaces $R(\beta,\omega)$ and $I(\beta,\omega)$ on the region $\mathfrak{D}=[\beta_{\min},\beta_{\max}]\times[\omega_{\min},\omega_{\max}]$, the equipotential contours are given by the intersections of the surfaces with the s-plane. Taking into account this geometric interpretation the $M(s)$ mapping based rootfinder can be summarized as follows:

Algorithm 1 Mapping based rootfinder

1 The region $\mathcal{D}=[\beta_{\min},\beta_{\max}]\times[\omega_{\min},\omega_{\max}]$, where the roots of $M(s)$ are to be computed, is defined.
2 With a chosen increment Δ_s, the region $\mathcal{D}$ is covered by the grid of nodes $d_{ij}=\beta_j\times\omega_i$, $i=1..\,|\omega_{\max}-\omega_{\min}|/\Delta_s$, $j=1..\,|\beta_{\max}-\beta_{\min}|/\Delta_s$ with stepwise incrementing co-ordinates β and ω.
3 For each node d_{ij}, the values of $R(\beta_i,\omega_j)$ and $I(\beta_i,\omega_j)$ are computed.
4 Using an contour plotting algorithm, the intersections of the surfaces R and I with the s-plane, i.e., contours $R(\beta,\omega)=0$ and $I(\beta,\omega)=0$, are mapped.
5 The intersection points of the contours $R(\beta,\omega)=0$ and $I(\beta,\omega)=0$ indicate the approximate positions of the poles.
6 The accuracy of the root approximations is enhanced by means of Newton's iteration method.

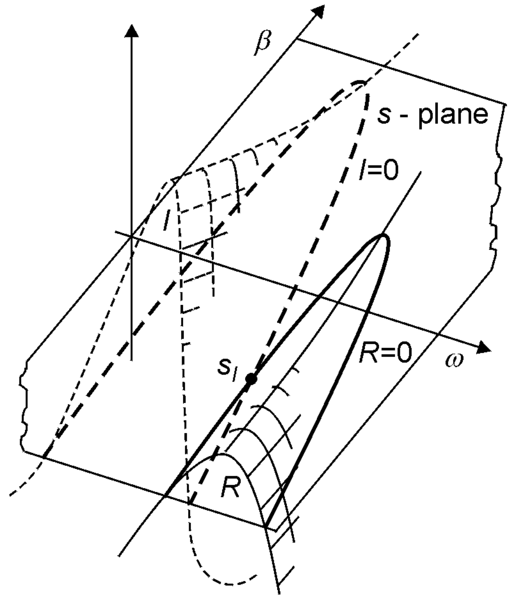

Fig 2 The principle of locating $M(s)$ roots, R=Re(M)=0 - solid, I=Im(M) = 0 - dashed

Since the contours $R(\beta,\omega)=0$ and $I(\beta,\omega)=0$ can be obtained analytically only for the most trivial quasipolynomials, e.g., $M(s)=s+\exp(-s)$, a numerical algorithm has to be used for that purpose. For mapping the contours, the algorithm known as level curve tracing algorithm, see, e.g., Cottafava and Le Moli, (1969), is used, which is available in Matlab as the function *contour*. Using this function, I have programmed function *aroots* (in Matlab) acomplishing Algorithm 1. The mapping based rootfinder may be used to compute both the poles and zeros of both retarded and neutral system with lumped and distributed delays, i.e., to compute the roots of quasipolynomial. Obviously, the algorithm can also be used to compute the roots of low degree polynomials and also the roots of exponential polynomials (which determine the essential spectrum of the neutral equations). In fact, the class of functions whose roots can be computed using the rootfinder is broader. Since the subject solved in this thesis deals with the analysis of TDS, the applicability of the mapping based rootfinder to further functions is not investigated. The applicability of the mapping based rootfinder is limited to the well-conditioned functions, (Wilkinson, 1984), which rather bounds the maximum degree of the (quasi)polynomials which can be analysed using the rootfinder (let us say up to degree 20). On the other hand, the anisochronic approach provides low order models of real plants as a rule. Thus the mapping based rootfinder is very valuable tool in the analysis of the dynamics of TDS.

4. Application of mapping based rootfinder in analysis and synthesis of time delay systems

In this chapter, the other three objectives are solved. Most of the methods designed in this chapter are based on the designed mapping based rootfinder given by Algorithm 1.

4.1 Features of first order anisochronic model with two delays

First, the features of the anisochronic first order model with two delays, given by transfer function

$$G(s)=\frac{y(s)}{u(s)}=\frac{K\exp(-s\tau)}{Ts+\exp(-s\eta)} \tag{23}$$

where K is steady state gain, T is time constant and τ is input time delay and η is the state delay are investigated. Thanks to its anisochronic structure, model (23) is quite universal and can be used to describe real plants which are usually described by higher order models, see, e.g., Vyhlídal and Zítek, (2001). In order to investigate the dynamics of (23), using the mapping based rootfinder, the positions of the dominant poles of (23) with respect to the values of the denominator parameters has been mapped, see Fig. 3.

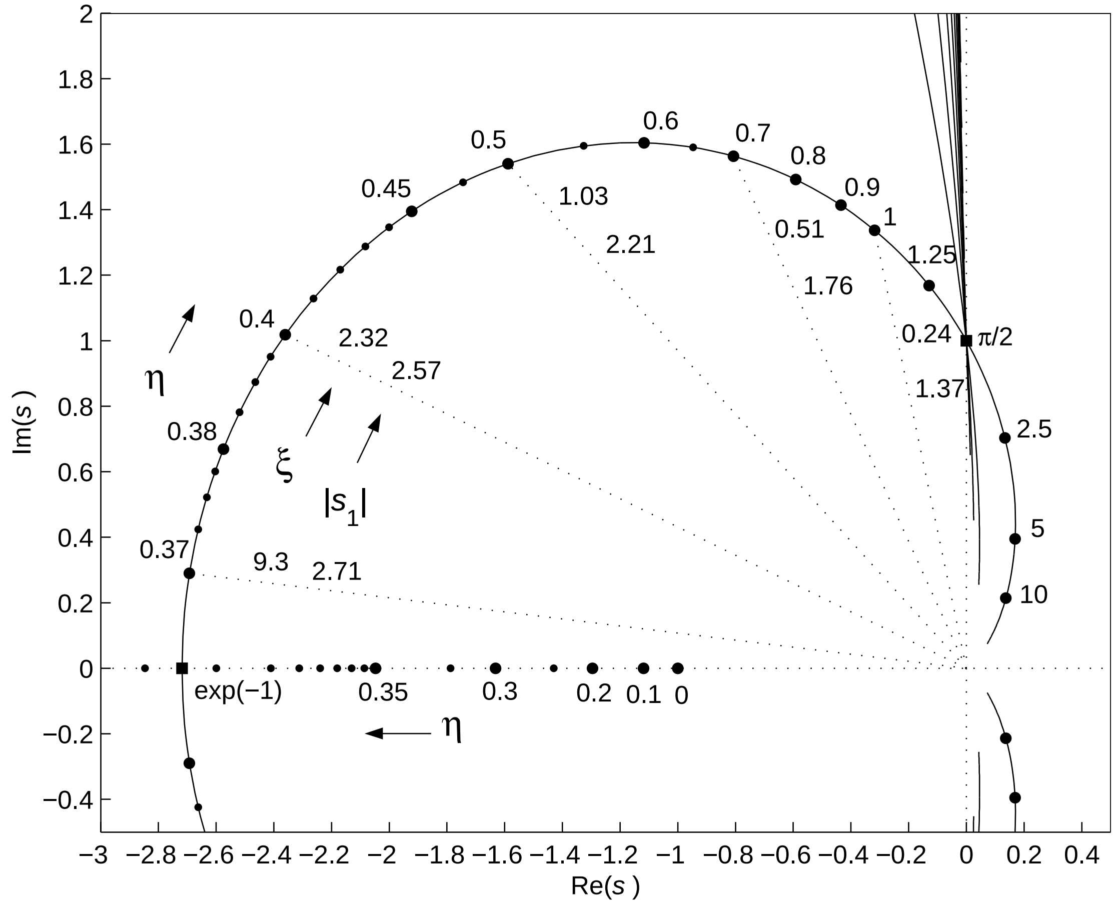

Fig. 3 The trajectories of the dominant pair of poles of (23) with respect to the value of η=η/T,
$\xi = |\beta|/\omega \quad s_{1,2} = \beta \pm \mathrm{j}\omega$

As can be seen, the position of the dominant couple of poles of (23) is determined by the ratio η/T. For $\eta/T \in (0, \exp(-1)]$ the dynamics of (23) is determined by the couple of real poles (the dynamics is well damped), while for $\eta/T > \exp(-1)$ the couple of dominant poles is complex conjugate (the dynamics is oscillatory with stable dynamics for $\eta/T \in (\exp(-1), \pi/2)$). The stability boundary is attained for $\eta/T = \pi/2$. Although the model is described by a single functional differential equation, the possibility to place the dominant couple of poles arbitrarily in the left half of the complex plane implies the universality of the model. It allows describing the dynamics of quite broad class of systems with damped as well as oscillatory dynamics. The universality of the first order anisochronic model can be further extended involving an exponential polynomial into the numerator of the transfer function. Consider the model

$$G(s) = \frac{K(1 - a\exp(-s\chi))\exp(-s\tau)}{Ts + \exp(-s\eta)} \tag{24}$$

As has been shown, the dominant couple of poles of (24) can be placed arbitrarily in the left half of the complex plane. By means of the parameters of the exponential polynomial $N(s) = 1 - a\exp(-s\chi)$, also the dominant zero can be assigned to (24). Choosing the parameters a and χ, the positions of the zeros, i.e., roots $s = \beta + \mathrm{j}\omega$ of $N(s)$ closest to the s-plane origin are given by

$$\beta = -\frac{1}{\chi}\ln\left|\frac{1}{a}\right| \tag{25}$$

$$\begin{matrix} \omega = \pm 2k\pi/\chi, & \text{if } a > 0 \\ \omega = \pm(2k+1)\pi/\chi, & \text{if } a < 0 \end{matrix}, \; k = 0,1,2,... \tag{26}$$

Thus, if a>0, the dominant zero of (24) is real given by (25) and if a<0, the dominant zero is complex conjugate with the real part (25) and the imaginary part given by (26) with k=0. To sum up, model (24) can be used to approximate the dynamics of the system by assigning the dominant pole and the dominant zero. Taking into account that the system dead time may be approximated by the numerator input delay τ, model (24) may be used to approximate the dynamics of a quite broad class of systems.

4.2 Evaluation of the significance of the poles of TDS

The significance of the poles (in stable system dynamics) is usually evaluated with respect to their distances from the imaginary axis, see e.g., Goodwin, (2001). Besides such a pole significance evaluation it is also reasonable evaluate the poles with respect to their magnitudes. It is difficult to claim whether the distance from the origin or the distance from the imaginary axis is more decisive in determining the particular pole significance. In this section, according to the third objective of the thesis, an original method for evaluating pole significance will be introduced based on the analysis of the modes of TDS.

Due to the generalized Heaviside series expansion, see, e.g., Angot, (1952), the transfer function $G(s)$ can be expanded into

$$G(s)=\frac{N(s)}{M(s)}=\sum_{i=1}^{\infty}\frac{N(\lambda_i)}{M'(\lambda_i)}\frac{1}{s-\lambda_i}=\sum_{i=1}^{\infty}\frac{R(\lambda_i)}{s-\lambda_i}=\sum_{i=1}^{\infty}H_i(s) \tag{27}$$

where $R(\lambda_i)$ are the residues corresponding to the poles λ_i, the functions $N(s)$ and $M(s)$ are the analytic functions, e.g., quasipolynomials, and $N(s)/M(s)$ has only single poles the number of which may be both finite and infinite.

If expansion (27) is applied to the transfer function of TDS, the system is expanded into the sum of infinitely many transfer functions, which can be either first order (corresponding to the real poles)

$$H_i(s)=\frac{R(\lambda_i)}{s-\lambda_i} \tag{28}$$

or second order (corresponding to the complex conjugate poles)

$$H_{i,i+1}(s)=\frac{R(\lambda_i)(s-\lambda_{i+1})+R(\lambda_{i+1})(s-\lambda_i)}{(s-\lambda_i)(s-\lambda_{i+1})}=\frac{2(\beta_{\mathrm{R}i}s-\beta_{\mathrm{R}i}\beta_i-\omega_{\mathrm{R}i}\omega_i)}{s^2-2\beta_i s+\beta_i^2+\omega_i^2} \tag{29}$$

where λ_{i+1} denotes here the complex conjugate pole to λ_i and $R(\lambda_{i,i+1})=\beta_{\mathrm{R}i}\pm \mathrm{j}\omega_{\mathrm{R}i}$ (for complex conjugate $\lambda_{i,i+1}$). Applying the inverse Laplace transform to (27), we obtain the weighting function of TDS as the sum of the infinitely many weighting functions of either (28) or (29). Evaluating the contribution of a particular weighting function to the weighting function of TDS may be used to evaluate the significance of the mode in the system dynamics. Performing the inverse Laplace transform to transfer functions (28) and (29), respectively, we obtain the weighting functions of the modes

$$h_i(t)=R(\lambda_i)\exp(\lambda_i t) \tag{30}$$

if λ_i are real poles and

$$h_{i,i+1}(t)=2(\beta_{\mathrm{R},i}\exp(\beta_i t)\cos(\omega_i t)-\omega_{\mathrm{R},i}\exp(\beta_i t)\sin(\omega_i t)) \tag{31}$$

if $\lambda_{i,i+1}$ are the complex conjugate pairs of poles, respectively. The significance evaluating criterion I suggest is based on evaluating the difference between the minimum and maximum of the weighting function

$$\begin{aligned}
&h_{\mathrm{e}i}=\left|h_{i\max}-h_{i\min}\right|\\
&h_{\mathrm{e}i}=\left|R(\lambda_i)\right| \text{ if } \lambda_i \text{ is real}\\
&h_{\mathrm{e}i}=\max\left\{\left|h_i(0)-h_i(t_{\mathrm{e}0})\right|, \left|h_i(t_{\mathrm{e}0})-h_i(t_{\mathrm{e}1})\right|\right\} \text{ if } \lambda_i \text{ is complex}
\end{aligned} \tag{32}$$

where $t=t_{\mathrm{e}i}+\pi/\omega$, k=0,1, 2, ..., are times at which $h_{i,i+1}(t)$ achieve the extrema given as the solutions of the equation

$$\tan(\omega_i t)=\frac{\beta_{\mathrm{R}i}\beta_i-\omega_{\mathrm{R}i}\omega_i}{\beta_{\mathrm{R}i}\omega_i+\omega_{\mathrm{R}i}\beta_i} \tag{33}$$

obtained from evaluating the first derivation of (31). The criterion is reliable and provides reliable evaluation of the pole significance of the distinct poles. However, if there are two poles in the spectrum that are very close to each other, the significance of the couple is to be evaluated instead of evaluating each pole separately. This rule is implied by the fact that as the poles are close to each other, their residues become very large (but with of the opposite signs). Thus if evaluated separately, criterion (32) unmatches the real significance of the poles. Their significance is evaluated as much higher than it really is.

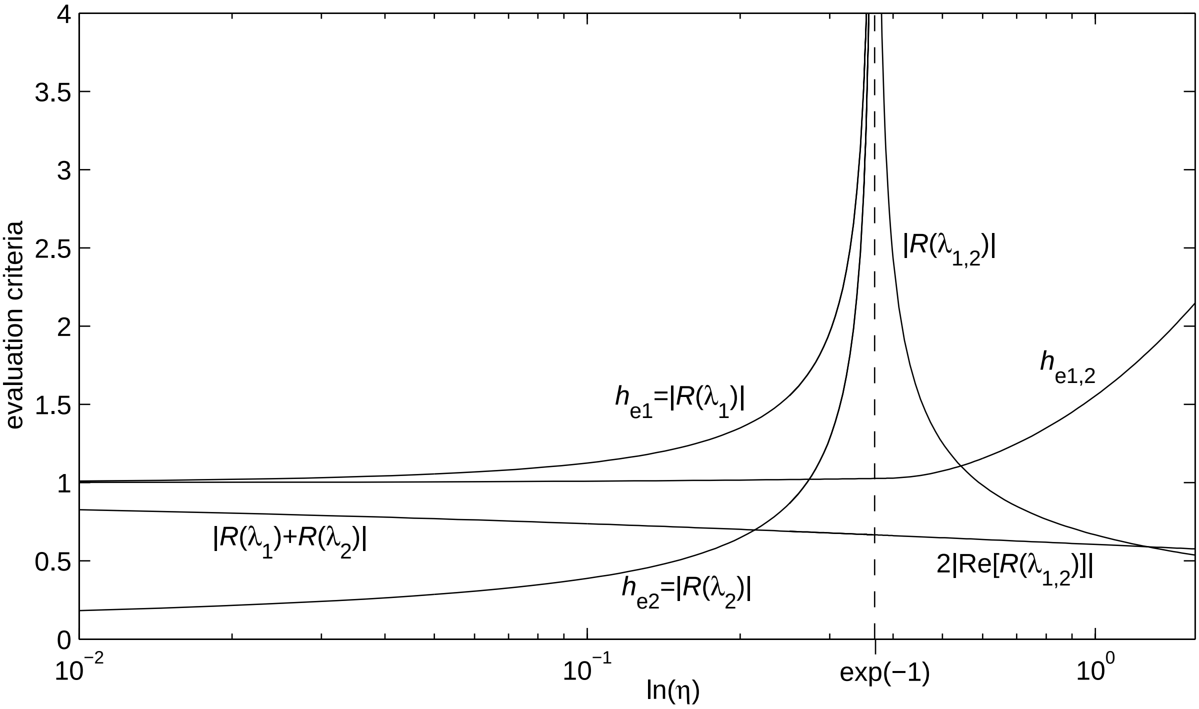

Fig. 4 The evaluating criteria of the significance of the dominant couple of poles of system (23)

However, if evaluated as a couple the weighting functions which are summed and the resultant weighting function is comparable with the weighting functions corresponding to the other poles. In Fig. 4, this feature is demonstrated on the evaluation of the poles of the system with the denominator $M(s) = s + \exp(-s)$, (compare with the values of the poles seen in Fig. 3). As can be seen, only $h_{e1,2}$ truly evaluates the contribution of the couple to the system dynamics.

According to the fact that Heaviside expansion (27) can be performed only if the poles are distinct, the criterion I have suggested cannot be directly used to evaluate the significance of multiple poles. The possible way of evaluating the contribution of a multiple pole to the dynamics consists in the following procedure. First, let the zero with the same multiplicity and the same value as the multiple pole be introduced into the numerator of the transfer function. Thus, the multiple pole is compensated. Secondly, let the compensated multiple pole be substituted by a group of distinct poles (the number of which is equal to the multiplicity of the pole) whose values are close to the value of the multiple pole. Finally, let the contribution of this group of poles is evaluated. If the poles that substitute the multiple pole are close to its value, their contribution to the dynamics is very close to the contribution of the multiple pole and the value of the criterion truly evaluates the significance of the multiple pole.

4.3 Gradient based state variable feedback control

In this section the main ideas of control method of TDS based on state variable feedback control are outlined and the features of the method are investigated. Consider coefficient feedback from the state variables (17) to control a retarded TDS. The characteristic equation of the feedback system is as follows

$$M(s,\mathbf{K}) = \det\left[s\,\mathbf{I} - \mathbf{A}(s) + \mathbf{B}(s)\mathbf{K} \right] = 0 \tag{34}$$

Suppose the original spectrum of poles, i.e., the eigenvalues of the system matrix $\mathbf{A}(s)$, is $\mathrm{Sp}(\mathbf{A}(s)) = \{\lambda_i\}, i = 1..\infty$ and the characteristic quasipolynomial of the original system is

$$M_0(s) = \det[s\mathbf{I} - \mathbf{A}(s)] \tag{35}$$

Closing feedback (17), the characteristic quasipolynomial of the system changes from (35) to the form of (34) with the new spectrum $\mathrm{Sp}(\mathbf{A}(s) - \mathbf{B}(s)\mathbf{K}) = \{\sigma_i\}, i = 1..\infty$. Since quasipolynomial (34) is linear with respect to **K** (Zítek and Vyhlídal, 2002), the following relationship holds between the original $M_0(s)$ and the feedback system quasipolynomial $M(s,\mathbf{K})$

$$M(s,\mathbf{K}) = M_0(s) + \sum_{j=1}^{n} \frac{\partial M(s,\mathbf{K})}{\partial K_j} K_j \tag{36}$$

Consider the original system has undesirable dynamics, i.e., the group of the most significant poles of the original system brings about too slow or less damped character of the dynamics. The aim of introducing the feedback from the state variables is to place the most significant system poles into the prescribed new positions $s = \sigma_i, i = 1,2,...,\ i_{\max} \le n$ (n is system order), which are chosen to endow the system with more favourable dynamics. For any prescription of σ_i the following relationship holds

$$M(\sigma_i, \mathbf{K}) = 0 = M_0(\sigma_i) + \sum_{j=1}^{r} K_j \left[\frac{\partial M(s, \mathbf{K})}{\partial K_j} \right]_{s=\sigma_i} \tag{37}$$

i.e., a set of linear algebraic equations with the unknown parameters $K_1, K_2, ..., K_n$. In fact, equations (37) corresponds only to the prescribed real poles σ_i. If a prescribed pole is complex $\sigma_i = \beta_i + \mathrm{j}\omega_i$, equation (37) has to be split into two equations $\mathrm{Re}(M(\sigma_i, \mathbf{K})) = 0$ and $\mathrm{Im}(M(\sigma_i, \mathbf{K})) = 0$. Evaluating the partial derivatives in (37) and substituting the prescribed poles into the equations, the following system of equations results

$$\mathbf{SK} = \mathbf{m} \tag{38}$$

$\mathbf{K} \in \mathrm{R}^r$ ($r \le n$), $\mathbf{S} \in \mathrm{R}^{q \times r}$ and $\mathbf{m} \in \mathrm{R}^q$ ($q \le n$) where

$$\mathbf{S} = \begin{bmatrix} S_{k,j} \\ S_{Rl,j} \\ S_{Il,j} \end{bmatrix},\ S_{k,j} = \left[\frac{\partial M(s, \mathbf{K})}{\partial K_j} \right]_{s=\sigma_k} \text{ for real } \sigma_k,$$
$$S_{Rl,j} = \mathrm{Re}\left(\left[\frac{\partial M(s, \mathbf{K})}{\partial K_j} \right]_{s=\sigma_l} \right),\ S_{Il,j} = \mathrm{Im}\left(\left[\frac{\partial M(s, \mathbf{K})}{\partial K_j} \right]_{s=\sigma_l} \right) \text{ for complex } \sigma_l \tag{39}$$

$$\mathbf{m} = \begin{bmatrix} m_k \\ m_{Rl} \\ m_{Il} \end{bmatrix},\ m_k = M_0(\sigma_k),\ m_{Rl} = \mathrm{Re}(M_0(\sigma_l)),\ m_{Il} = \mathrm{Im}(M_0(\sigma_l)) \tag{40}$$

where $k = 1..q_{\mathrm{r}}$, q_{r} is the number of prescribed real poles, $l = 1..q_{\mathrm{c}}$, q_{c} is the number of prescribed imaginary poles, $q = q_{\mathrm{r}} + 2q_{\mathrm{c}}$, and j=1..r, r is the number of feedback loops from the state variables.

Obviously, the maximum number of the poles that might be prescribed using the state variable feedback control is equal to the number of the state variables, i.e., to the system order n. Thus, having $q = n$, the system of equations (38) may be solved as $\mathbf{K} = \mathbf{S}^{-1}\mathbf{m}$ if the matrix $\mathbf{S}$ is non-sigular. However, more numerically stable techniques of solving system of equation (38) are the iterative methods, e.g., Gauss-Seidel method.

In general, there may be less than n significant poles with undesirable positions in the spectra of the original system. Provided that the other poles are much farther to the left from the imaginary axis, it is reasonable to prescribe new positions only to these poles with undesirable positions. One possibility to solve the task for $q < n$ consists in using only $r = q$ feedback loops, which reduce the problem to solving set of equations (38). Another possibility, which is likely to result in more robust dynamics of the feedback system, consists in using all the feedback loops which are available. To obtain the feedback coefficients, set of underdetermined equations (4.68), $r>q$, is to be solved. Using the Moore-Penrose inverse $\mathbf{S}^+$ of $\mathbf{S}$ (Ben-Israeland Greville, 1977), the feedback coefficients are given by

$$\mathbf{K} = \mathbf{S}^+\mathbf{m} \tag{41}$$

The Moore-Penrose generalized matrix inverse is a unique matrix pseudoinverse, which provides the solution with the minimal norm $\| \mathbf{K} \|_2$. Apparently the desired eigenvalue positions are to be prescribed with respect to λ_i, $i = 1,2,...$ constituting the group of the most significant system poles. It is of little sense to assign the insignificant system poles because they cannot affect the actual system behaviour. The crucial problem of pole assignment in TDS is the following. Although equation set (41) may be solved for arbitrary set of given $\sigma_i, i = 1,2,...,n$, in fact the region where these prescribed values may be taken from is rather restricted. Obviously, the prescribed $\sigma_i, i = 1,2,...,n$ have to correspond to the eigenvalues λ_i with the largest values of the criterion h_{e_i}. Obviously, the set of n eigenvalues is only a little part of the whole spectrum. Thus an infinite set of the rest of eigenvalues is placed spontaneously. To get the prescribed σ_i actually determining the system dynamics, it is necessary that the assigned eigenvalues constitute the set of most significant poles of the system dynamics being designed. Basically, it means that σ_i must not be prescribed too fast, i.e. too far to the left with respect to the original positions of the poles. If such too fast eigenvalue is prescribed, the consequence is that one or more of spontaneously placed eigenvalues takes over the role of the significant poles in the new spectrum. Often such spontaneously placed pole causes the instability of the system. To avoid safely the case of such a pole placement failure, it is necessary to try repeatedly a sequence of the prescribed new positions of the dominant system poles of the stepwise increasing sizes. The critical size of this shifting is when firstly a spontaneously placed dominant pole with undesirable position appears.

An alternative approach to the direct pole placement is the continuous pole placement introduced by Michiels, et al., (2002). Prescribing small shifting of the poles from the current positions, the increments of the feedback gain coefficients in $\Delta\mathbf{K}$ can be computed on the basis of the sensitivity matrix. As will be shown, such a continuous shifting can also be performed using the gradient based feedback control design. As has been shown, if the prescribed poles are complex, equation (35) has to be split into real and imaginary parts. Substituting $s = \beta + \mathrm{j}\omega$ into (35) yields

$$M(\beta + \mathrm{j}\omega) = \det\left[(\beta + \mathrm{j}\omega)\mathbf{I} - \mathbf{A}(\beta + \mathrm{j}\omega) + \mathbf{B}(\beta + \mathrm{j}\omega)\mathbf{K}\right] = R(\beta, \omega, \mathbf{K}) + \mathrm{j}I(\beta, \omega, \mathbf{K}) \tag{42}$$

Prescribing only the real parts of the complex poles, i.e., β_i, we have the following equations

$$R(\beta_i, \omega_i, \mathbf{K}) = 0 \tag{43}$$

$$I(\beta_i, \omega_i, \mathbf{K}) = 0 \tag{44}$$

for each of the prescribed β_i with the variables $K_1, K_2, ..., K_r$ and ω_i to be computed. Unlike equation (35), equations (43) and (44) are linear neither with respect to $K_1, K_2, ..., K_r$ nor with respect to ω_i (since ω_i are considered as the unknown variables). This non-linearity has the inconvenient consequence of loosing the possibility to place the poles arbitrarily. Consider, the actual setting of the feedback coefficients is $\tilde{\mathbf{K}}$ and the complex poles $\lambda_i = \beta_i + \mathrm{j}\omega_i$ correspond to this setting. Suppose we displace the real parts of the poles $\beta_i \to \beta_i + \Delta\beta_i, i = 1,2,...,q_\mathrm{c}$ (q_c is the number of prescribed complex poles). Provided that $\Delta\beta_i$ are small, approximate values of $\Delta\omega_i$ and ΔK_j can be obtained as the solutions of the following set of equations

$$R(\beta_i + \Delta\beta_i, \omega_i, \mathbf{K}) + \sum_{j=1}^{r} \Delta K_j \left[\frac{\partial R(\beta, \omega, \mathbf{K})}{\partial K_j}\right]_{\substack{\beta = \beta_i + \Delta\sigma \\ \omega = \omega_i \\ \mathbf{K} = \tilde{\mathbf{K}}}} + \Delta\omega_i \left[\frac{\partial R(\beta, \omega, \mathbf{K})}{\partial K_j}\right]_{\substack{\beta = \beta_i + \Delta\sigma \\ \omega = \omega_i \\ \mathbf{K} = \tilde{\mathbf{K}}}} = 0 \tag{45}$$

$$I(\beta_i + \Delta\beta_i, \omega_i, \mathbf{K}) + \sum_{j=1}^{r} \Delta K_j \left[\frac{\partial I(\beta, \omega, \mathbf{K})}{\partial K_j}\right]_{\substack{\beta = \beta_i + \Delta\sigma \\ \omega = \omega_i \\ \mathbf{K} = \tilde{\mathbf{K}}}} + \Delta\omega_i \left[\frac{\partial I(\beta, \omega, \mathbf{K})}{\partial K_j}\right]_{\substack{\beta = \beta_i + \Delta\sigma \\ \omega = \omega_i \\ \mathbf{K} = \tilde{\mathbf{K}}}} = 0 \tag{46}$$

which result from linearizing (43) and (44), respectively. Thus, analogously to (38), we can write the system of equations

$$\mathbf{S}\begin{bmatrix}\Delta\mathbf{K} \\ \Delta\boldsymbol{\omega}\end{bmatrix} = \mathbf{m} \tag{47}$$

$\Delta\mathbf{K} \in \mathrm{R}^r$, $\Delta\boldsymbol{\omega} \in \mathrm{R}^{q_\mathrm{c}}$, $\mathbf{S} \in \mathrm{R}^{q \times (r + q_\mathrm{c})}$ and $\mathbf{m} \in \mathrm{R}^q$

$$\begin{bmatrix}\Delta\mathbf{K} \\ \Delta\boldsymbol{\omega}\end{bmatrix} = \left[\Delta K_1\, \Delta K_2 \dots \Delta K_r\, \Delta\omega_1\, \Delta\omega_2 \dots \Delta\omega_{q_\mathrm{c}}\right]^\mathrm{T} \tag{48}$$

$$\mathbf{S}=\begin{bmatrix} S_{1,1} & S_{1,2} & \dots & S_{1,r} & 0 & 0 & \dots & 0 \\ S_{2,1} & S_{2,2} & \dots & S_{2,r} & 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ S_{p_r,1} & S_{p_r,2} & \dots & S_{p_r,r} & 0 & 0 & \dots & 0 \\ S_{R1,1} & S_{R1,2} & \dots & S_{R1,r} & S_{R,\omega_1} & 0 & \dots & 0 \\ S_{R1,1} & S_{R1,1} & \dots & S_{R1,r} & 0 & S_{R,\omega_2} & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & \ddots & \vdots \\ S_{R_{q_c},1} & S_{R_{q_c},2} & \dots & S_{R_{q_c},r} & 0 & 0 & 0 & S_{R,\omega_{q_c}} \\ S_{I1,1} & S_{I1,1} & \dots & S_{I1,1} & S_{I,\omega_1} & 0 & \dots & 0 \\ S_{I2,1} & S_{I2,2} & \dots & S_{I2,r} & 0 & S_{I,\omega_2} & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{I_{q_c},1} & S_{I_{q_c},2} & \dots & S_{I_{q_c},r} & 0 & 0 & 0 & S_{I,\omega_{q_c}} \end{bmatrix} \tag{49}$$

$$S_{k,j}=\left[\frac{\partial M(s,\mathbf{K})}{\partial K_j}\right]_{\substack{s=\sigma_k+\Delta\sigma \\ \mathbf{K}=\widetilde{\mathbf{K}}}}, S_{Rl,j}=\left[\frac{\partial R(\beta,\omega,\mathbf{K})}{\partial K_j}\right]_{\substack{\beta=\beta_l+\Delta\sigma \\ \omega=\omega_l \\ \mathbf{K}=\widetilde{\mathbf{K}}}}, S_{Il,j}=\left[\frac{\partial I(\beta,\omega,\mathbf{K})}{\partial K_j}\right]_{\substack{\beta=\beta_l+\Delta\sigma \\ \omega=\omega_l \\ \mathbf{K}=\widetilde{\mathbf{K}}}}$$

$$S_{R,\omega_l}=\left[\frac{\partial R(\beta,\omega,\mathbf{K})}{\partial \omega}\right]_{\substack{\beta=\beta_l+\Delta\sigma \\ \omega=\omega_l \\ \mathbf{K}=\widetilde{\mathbf{K}}}}, S_{I},\omega_l=\left[\frac{\partial I(\beta,\omega,\mathbf{K})}{\partial \omega}\right]_{\substack{\beta=\beta_l+\Delta\sigma \\ \omega=\omega_l \\ \mathbf{K}=\widetilde{\mathbf{K}}}}$$

$$\mathbf{m}=\begin{bmatrix} m_k \\ m_{Rl} \\ m_{Il} \end{bmatrix},\ m_k=M(\sigma_l+\Delta\sigma,\widetilde{\mathbf{K}}),\ m_{Rl}=R(\beta_l+\Delta\sigma,\omega_l,\widetilde{\mathbf{K}}),\ m_{Il}=I(\beta_l+\Delta\sigma,\omega_l,\widetilde{\mathbf{K}}) \tag{50}$$

where $k=1..q_{\mathrm{r}}$, q_{r} is the number of prescribed displacements of the real poles σ_k, $k=1..q_{\mathrm{c}}$, q_{c} is the number of prescribed displacements of the complex poles, i.e., the real parts of the poles, $q=q_{\mathrm{r}}+q_{\mathrm{c}}$, and j=1..r, r is the number of feedback loops from the state variables. Thus prescribing the sufficiently small displacements $\Delta\sigma$ from the current right-most poles σ_k and $\beta_l+\mathrm{j}\omega_l$, the feedback increments ΔK_j and the displacements in imaginary parts of the complex poles $\Delta\omega_l$ can be computed from set of equations (47). If $r=q$ the set of equations can be solved in a classical method for solving the linear system of equations. If $r<q$ the underdetermined set of equations can be solved using the Moore-Penrose inversion

$$\begin{bmatrix} \Delta\mathbf{K} \\ \Delta\boldsymbol{\omega} \end{bmatrix}=\mathbf{S}^{+}\mathbf{m} \tag{51}$$

Thus, analogously to the algorithm for continuous pole placement based on the sensitivity functions introduced by Michiels, et al., (2002), we can write the algorithm for the rightmost pole shifting using described gradient based pole placement method.

Algorithm 2 Continuous pole placement based on gradient based method

A. Start with $q=1$
B. Compute the rightmost system poles using the mapping based rootfinder given by Algorithm 1
C. Assemble matrices (49) and (50) for system of equations (51)
D. Move q rightmost poles for which set of equations (51) has been assembled in direction to the left and find solution of (51).
E. Monitor the positions of the rightmost poles of the system with the computed feedback settings. If necessary, increase the number of controlled poles q. Stop when stability is reached or when the available degrees of freedom of the controller do not allow sup(Re(λ_i)), $i=1..\infty$ to be further reduced. In the other case, go to step B.

Unlike the continuous pole placement algorithm introduced in Michiels, et al., (2002), using the method described above, we obtain not only the changes of the coefficients ΔK_j but also the displacements of the imaginary parts of the poles $\Delta\omega_l$ which are shifted. This fact may be useful in the task of accelerating the continuous pole placement procedure. It is also important to note that from the numerical stability point of view, there should always be minimum distances between the neighbouring poles being controlled. If two of these poles are too close to each other, equation (51) becomes ill-conditioned. In the limit case of two controlled poles are identical, matrix (49) becomes singular. Even though the Moore-Penrose inversion of such a singular matrix exists, Algorithm 2 is likely to brake down. Unlike the algorithm presented by Michiels, et al., (2002), the gradient based feedback control offers the possibility to prescribe multiple poles. However, from the numerical point of view, keeping the poles distinct is safer. Moreover, the applicability of the Algorithm 2, as well as the direct pole placement, can be applied to a broader class of systems than the algorithm suggested in Michiels, et al., (2002), i.e., to both retarded and neutral systems with both lumped and distributed delays. On the other hand, since the continuous pole placement given by Algorithm 2 is based on the characteristic function, its applicability is restricted to the lower order TDS.

5. Real plant application example

All the dynamics analysis and control design methods introduced or investigated in the thesis, i.e., mapping based rootfinder, pole significance evaluating criterion and the methods for pole placement, have been tested on the model of laboratory plant heating system. The heating system (see its scheme in Fig. 5) consists of two heating circuits with the circulation of the heat medium (water) accomplished by two pumps (one in each circuit). The heat source of the system is an electric heater, located in the primary circuit. The heat exchange between the two circuits, which is controlled by the mixing valve, takes place in the multi-plate heating exchanger. The last important component of the system is an air-water cooler located in the secondary circuit. As can be seen in Fig. 5, the components of the system are connected by the piping lines that provide the most important delays in the system. Using the anisochronic approach, all the substantial parts of the system have been described using the first order anisochronic model. In this way obtained model is given by the following description

$$s\mathbf{x}(s) = \mathbf{A}(s)\mathbf{x}(s) + \mathbf{B}(s)\Delta\vartheta_{\mathrm{h,set}}(s) \tag{52}$$

where $\mathbf{x}(s) = [\Delta\vartheta_{\mathrm{h}}(s)\ \ \Delta\vartheta_{\mathrm{a}}(s)\ \ \Delta\vartheta_{\mathrm{d}}(s)\ \ \Delta\vartheta_{\mathrm{c}}(s)]^T$ is the vector of the state variables, where $\vartheta_{\mathrm{h}}(s),\ \vartheta_{\mathrm{a}}(s),\ \vartheta_{\mathrm{d}}(s),\ \vartheta_{\mathrm{c}}(s)$ are the temperatures measured on the laboratory system

$$\mathbf{A}(s) = \begin{bmatrix} \dfrac{-\exp(-\eta_{\mathrm{h}}s)}{T_{\mathrm{h}}} & \dfrac{K_{\mathrm{b}}\exp(-\tau_{\mathrm{b}}s)}{T_{\mathrm{h}}} & 0 & 0 \\ \dfrac{K_{\mathrm{a}}}{T_{\mathrm{a}}} & \dfrac{-(1+0.5K_{\mathrm{a}}(1+q))}{T_{\mathrm{a}}} & 0 & \dfrac{(1-0.5K_{\mathrm{a}}(1-q))\exp(-\tau_{\mathrm{e}}s)}{T_{\mathrm{a}}} \\ 0 & \dfrac{K_{\mathrm{d}}\exp(-\tau_{\mathrm{d}}s)}{T_{\mathrm{d}}} & \dfrac{-1}{T_{\mathrm{d}}} & 0 \\ 0 & 0 & \dfrac{K_{\mathrm{c}}\exp(-\tau_{\mathrm{c}}s)}{T_{\mathrm{c}}} & \dfrac{-\exp(-\eta_{\mathrm{c}}s)}{T_{\mathrm{c}}} \end{bmatrix}$$

$$\mathbf{B}(s) = \begin{bmatrix} \dfrac{K_{\mathrm{u}}\exp(-\tau_{\mathrm{u}})}{T_{\mathrm{h}}} & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \quad \mathbf{C} = [0001]\ (y(t) = \vartheta_{\mathrm{c}}(t))$$

The parameters that assure a quite good approximation of the system step response performed in the vicinity of the operational point for which the model is assumed to approximate the system dynamics are the following: $T_{\mathrm{h}} = 14\mathrm{s}$, $K_{\mathrm{b}} = 0.24$, $K_{\mathrm{u}} = 0.39$, $\eta_{\mathrm{h}} = 6.5\mathrm{s}$, $\tau_{\mathrm{b}} = 40\mathrm{s}$, $\tau_{\mathrm{u}} = 13.2\mathrm{s}$, $T_{\mathrm{a}} = 3\mathrm{s}$, $K_{\mathrm{a}} = 1$, $\tau_{\mathrm{e}} = 13\mathrm{s}$, $q = 1$ ($m_1 = m_2 =$ 0.08 m^3/hour), $T_{\mathrm{d}} = 3\mathrm{s}$, $K_{\mathrm{d}} = 0.94$, $\tau_{\mathrm{d}} = 18\mathrm{s}$, $T_{\mathrm{c}} = 25\mathrm{s}$, $K_{\mathrm{c}} = 0.81$, $\eta_{\mathrm{c}} = 9.2\mathrm{s}$, $\tau_{\mathrm{c}} = 2.8\mathrm{s}$.

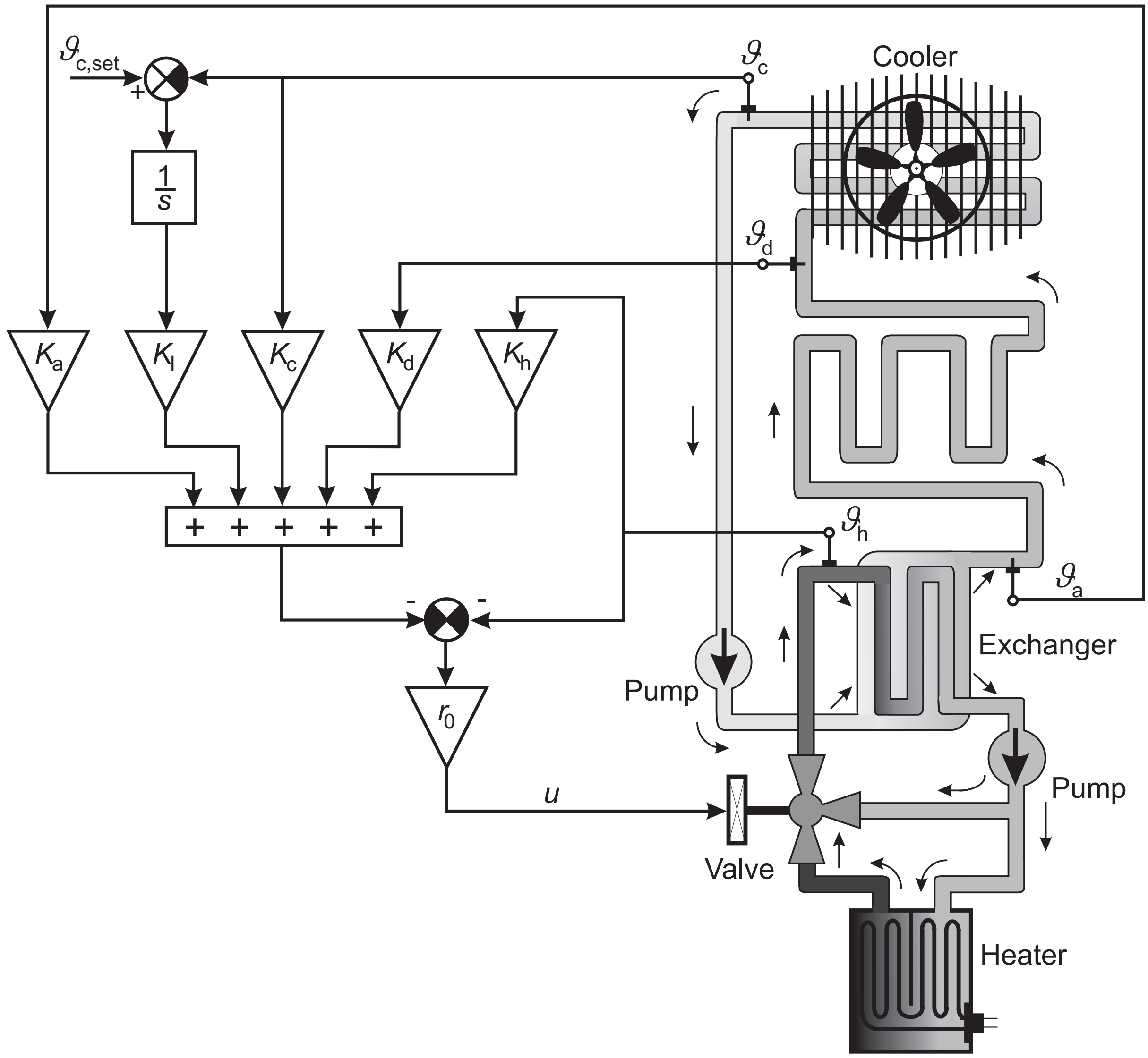

Fig. 5 Scheme of the state variable feedback control of the laboratory heating system

Having the linear model of the laboratory heating system in the form of retarded system, let us analyse the modes of the system dynamics. First of all, let us transform model (52) into the form of the transfer function

$$G(s) = G_{\mathrm{f}}(s)\exp(-s\tau) = \frac{N(s)}{M(s)}\exp(-s\tau) \tag{53}$$

According to (9) and (10) the numerator and denominator of (53) result in

$$N = 4.04\,10^{-5} \tag{54}$$

$$M(s) = s^4 + \sum_{i=0}^{3} Q_i(s)s^i \tag{55}$$

where $Q_i(s)$ are in the forms of sums of exponential functions and the input delay $\tau = 34\mathrm{s}$. Since the numerator part N of (54) is constant, the system does not have any zeros. The character of the input-output dynamics is given only by poles of (53), i.e., the roots of $M(s)$ given by (55). Using the quasipolynomial mapping based rootfinder given by Algorithm 3.1, the spectrum of system poles can be seen in Fig. 6. In Tab. 1, we can see the values of 34 system poles closest to the s-plane origin with the corresponding values of the residues applied to expanded $G_{\mathrm{f}}(s)$ using (27). The poles are ordered with respect to their significance that has been evaluated using criterion (32). As can be seen in Tab. 1, according to the criterion, the most significant poles of the system are $\lambda_{1,2}$ and λ_3. Also the following two couples of poles, i.e., $\lambda_{4,5}$ and $\lambda_{6,7}$, may be considered as the significant according to the chosen criterion (but less significant than $\lambda_{1,2}$ and λ_3).

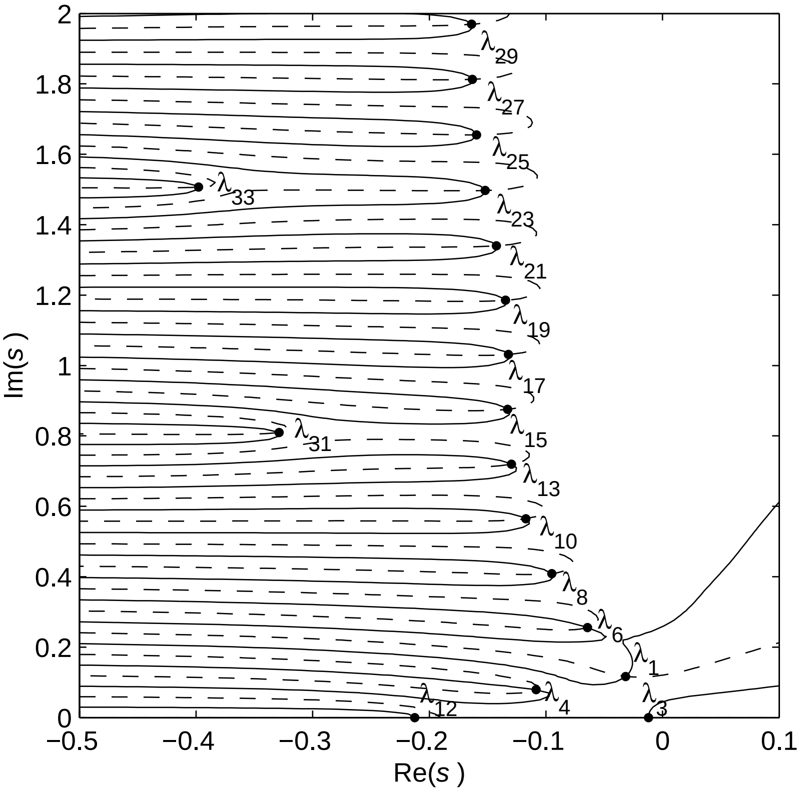

Fig. 6 Mapping quasipolynomial (55), Re($M(s)$) - solid, Im($M(s)$) - dashed. Roots of the quasipolynomial given as the intersections of the contours.

Tab. 1 The poles of system (52) and the values of the significance evaluating criterion

i	λ_i [s^{-1}]	h_{ei}
1	-0.0316 + 0.1167j	4.921 10^{-3}
3	-0.0121	3.801 10^{-3}
4	-0.1083 + 0.0791j	7.928 10^{-4}
6	-0.0643 + 0.2553j	3.817 10^{-4}
8	-0.0951 + 0.4088j	5.170 10^{-5}
10	-0.1171 + 0.5648j	1.443 10^{-5}
12	-0.2125	9.317 10^{-6}
13	-0.1295 + 0.7197j	7.607 10^{-6}
15	-0.1327 + 0.8755j	4.858 10^{-6}
17	-0.1322 + 1.0311j	2.675 10^{-6}
19	-0.1347 + 1.1852j	1.602 10^{-6}
21	-0.1425 + 1.3400j	1.038 10^{-6}
23	-0.1521 + 1.4972j	6.934 10^{-7}
25	-0.1595 + 1.6553j	4.133 10^{-7}
27	-0.1630 + 1.8128j	2.687 10^{-7}
29	-0.1636 + 1.9692j	2.190 10^{-7}
31	-0.3288 + 0.8097j	5.906 10^{-9}
33	-0.3979 + 1.5064j	1.172 10^{-10}

In Fig. 7 we can see the step responses of $G_f(s)$ and the step responses of the transfer functions corresponding to the modes of the system $H_{1,2}(s)$, $H_3(s)$, $H_{4,5}(s)$ and $H_{6,7}(s)$. As can be seen in Fig. 7, the correspondence of the step responses of $G_f(s)$ and $H_{1,2}(s)+H_3(s)$ confirms the determining roles of the poles $\lambda_{1,2}$ and λ_3 in the system dynamics.

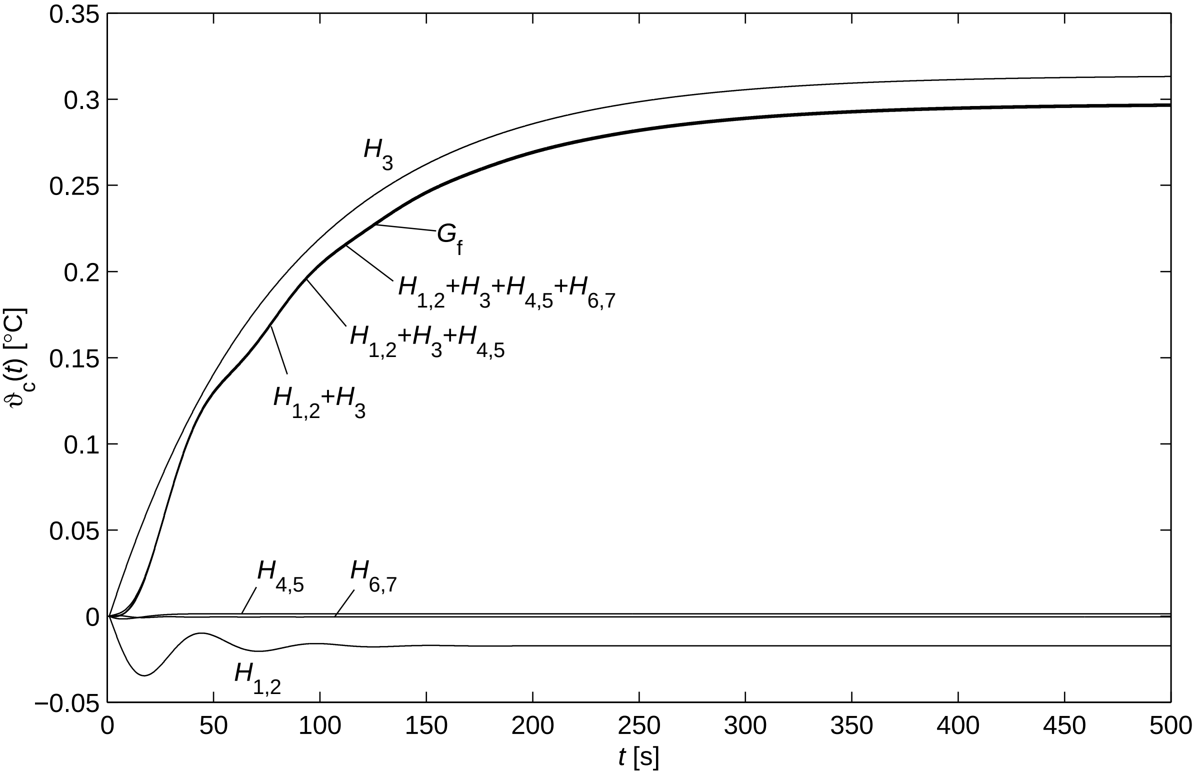

Fig. 7 Step responses of the system given by transfer function $G_f(s)$, see (53), and of the transfer functions $H_i(s)$ and their sums approximating $G_f(s)$. $\Delta\vartheta_{h,set}(1) = 1^\circ C$

The second task solved in this chapter is the control design of the system output temperature $\vartheta_{\mathrm{c}}(t)$ using the feedback loops from the state variables. In order to ascertain zero control error at the steady states of the feedback system, the additional state variable is introduced

$$\frac{dI(t)}{dt} = \Delta\vartheta_{\mathrm{c,set}}(t) - \Delta\vartheta_{\mathrm{c}}(t) \tag{56}$$

where $\vartheta_{\mathrm{c,set}}(t)$ is the set-point (desired) value of $\vartheta_{\mathrm{c}}(t)$. Thus the feedback control is accomplished by

$$\Delta\vartheta_{\mathrm{h,set}}(t) = -[K_1 \quad K_2 \quad K_3 \quad K_4 \quad K_5][\Delta\vartheta_{\mathrm{h}}(t) \quad \Delta\vartheta_{\mathrm{a}}(t) \quad \Delta\vartheta_{\mathrm{d}}(t) \quad \Delta\vartheta_{\mathrm{c}}(t) \quad I(t)]^{\mathrm{T}} \tag{57}$$

Since (52) and (56) are in a series linkage, additional state equation introduces only a single pole to the system dynamics located in the *s*-plane origin $\lambda_{35} = 0$ s^{-1}. First, in order to move the rightmost poles as to the left as possible, the continuous pole placement method given by Algorithm 2 is applied. The result of continuous shifting of the rightmost poles to the left is seen in Fig. 8. As can be seen, first, only the pole λ_{35} is being shifted. Gradually, the number of poles that are shifted is increased up to five, keeping the distances between the poles to assure the robust numerical computation (the number of controlled poles cannot be higher than the number of feedback loops). The pole placement procedure stops as the five poles being controlled get close to another pole. The resultant feedback gains (the evolution of the gains during the continuous pole placement procedure is seen in Fig. 9) are the following $\mathbf{K} = [0.245 \quad 1.101 \quad 3.181 \quad 3.348 \quad -0.119]$ corresponding to the rightmost poles $\lambda_{35} = -0.0413\mathrm{s}^{-1}$, $\lambda_3 = -0.0502\mathrm{s}^{-1}$, $\lambda_{12} = -0.0594\mathrm{s}^{-1}$, $\lambda_{1,2} = -0.0683 \pm 0.1278\mathrm{j}$ s^{-1}.

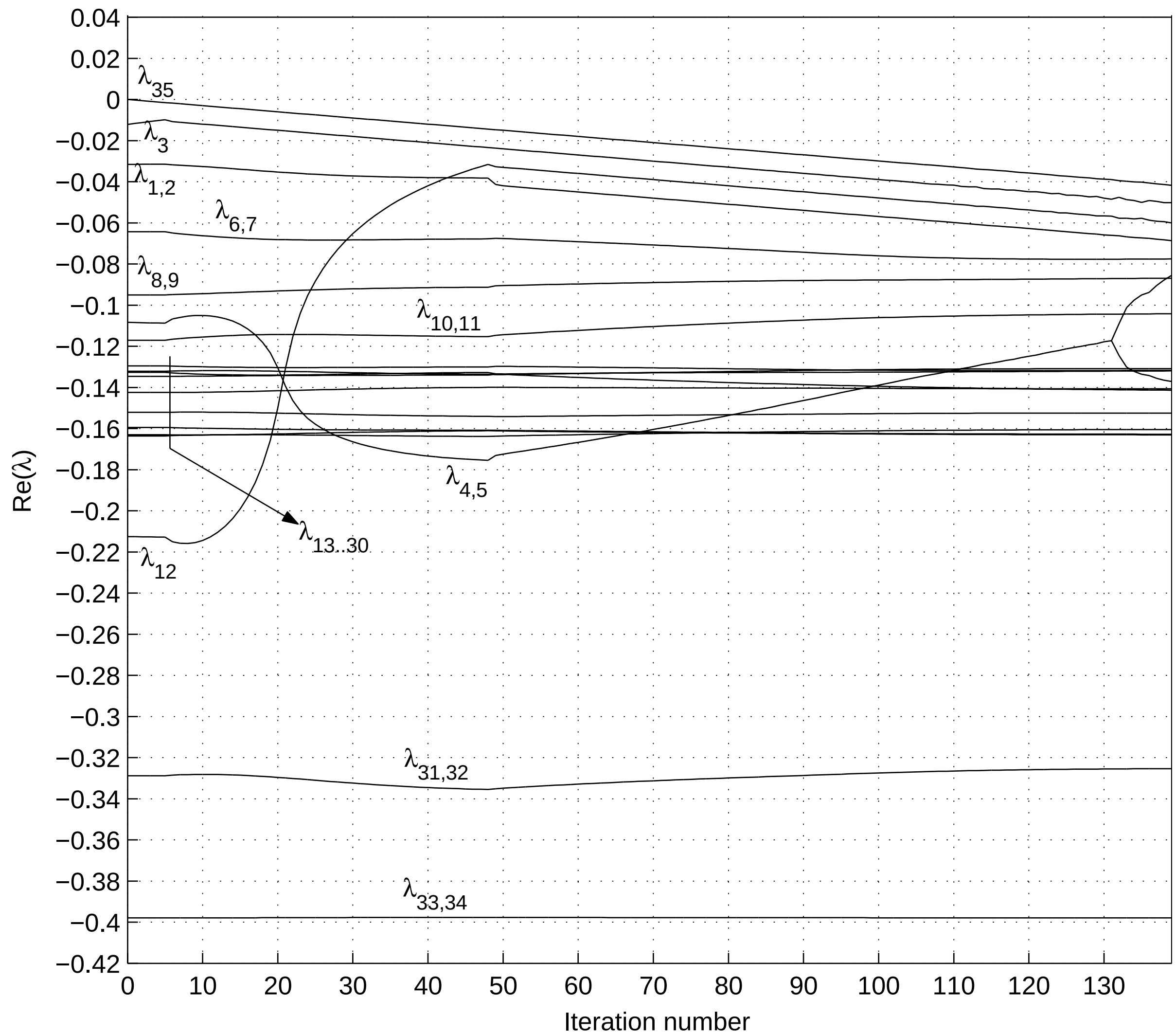

Fig. 8 The evolution of the real parts of the poles during the continuous pole placement given by Algorithm 2 applied to laboratory plant model given by (52)

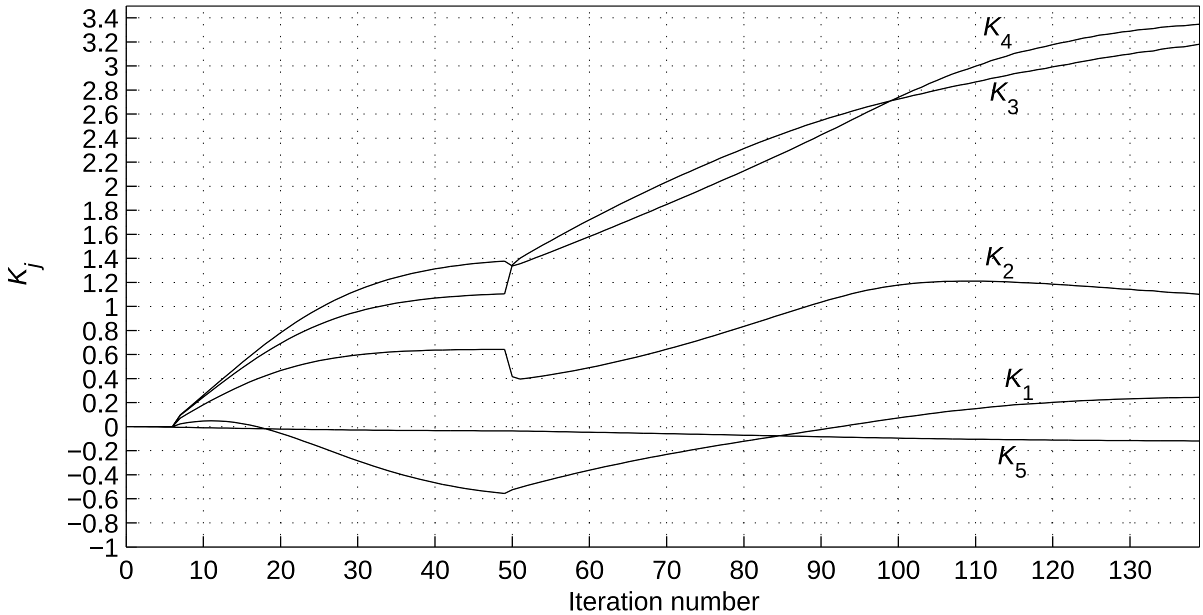

Fig. 9 The evolution of the feedback gains during the continuous pole placement

Since three of the poles are real, the dynamics of the system are rather overdamped. Thus, to further improve the system dynamics (following the requirement to achieve well damped and fast dynamics) the direct pole placement method is applied. The results of the procedure of direct prescribing the poles are seen in Tab. 2. For the comparison of the set-point responses with the designed feedback settings see Fig. 10

Tab. 2 Sets of prescribed system poles and the resultant feedback gains

k	Prescribed roots [s^{-1}]	$\mathbf{K}_k$
1	5×-0.072	$[0.277 \;\; 2.028 \;\; 3.841 \;\; 3.871 \;\; -0.159]$
2	$3\times -0.07, -0.03+0.06\mathrm{j}$	$[0.263 \;\; 2.230 \;\; 5.523 \;\; 4.513 \;\; -0.225]$
3	$-0.07, 2\times(-0.03+0.06\mathrm{j})$	$[-0.298 \;\; 4.091 \;\; 7.690 \;\; 4.576 \;\; -0.352]$
	c.p.p	$[0.245 \;\; 1.101 \;\; 3.181 \;\; 3.348 \;\; -0.119]$

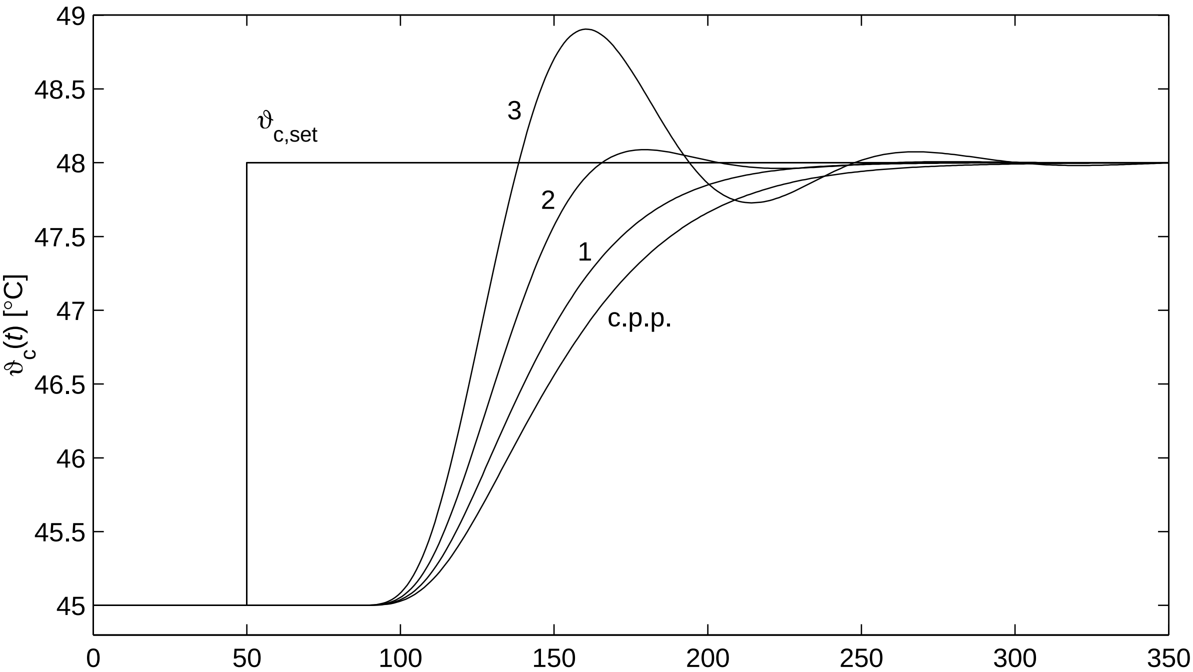

Fig. 10 Comparison of the set-point responses of the laboratory system with the feedback from the state variables with various settings of the feedback coefficients, c.p.p. - the feedback coefficients resulted from the continuous pole placement, k = 1, 2, 3, the feedback coefficient settings according to Tab. 2

As has been shown, using continuous pole placement method to shift the poles as much to the left as possible is convenient. Even though the feedback system dynamics given as the result of the continuous pole placement is not optimal as a rule, it provides very good starting pole distribution for applying direct pole placement. It would be very difficult to achieve the adequately good result starting the direct prescribing the poles from the pole distribution of the system without the feedback, seen Fig. 6.

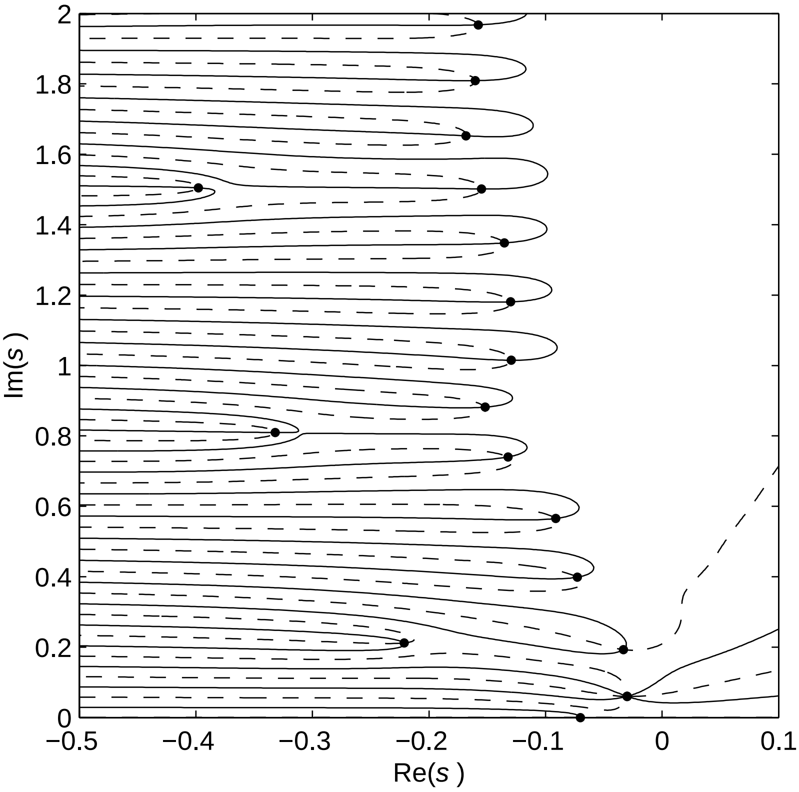

Fig. 11 Poles of the closed feedback system with the feedback coefficients, seen in Tab. 2, k = 3

In Fig. 12, we can see the comparison of the simulated set-point response of the model with the set-point response performed and measured on the laboratory heating system with the coefficient feedback gains taken from Tab. 2, k=3. The distribution of the dominant poles of the feedback system with this setting are seen in Fig. 11. Even though the parameters of the model of the heating system has been identified only on the basis of one measured step response performed in the operational point, the set-point response performed in the vicinity of the operational point is very close to the simulated set-point response. As can be seen, not only the dynamics of the system output temperature is modelled well, but also the responses of the other temperatures show very good equivalence between the model and the system dynamics in the vicinity of the operational point for which the linear plant anisochronic model is valid.

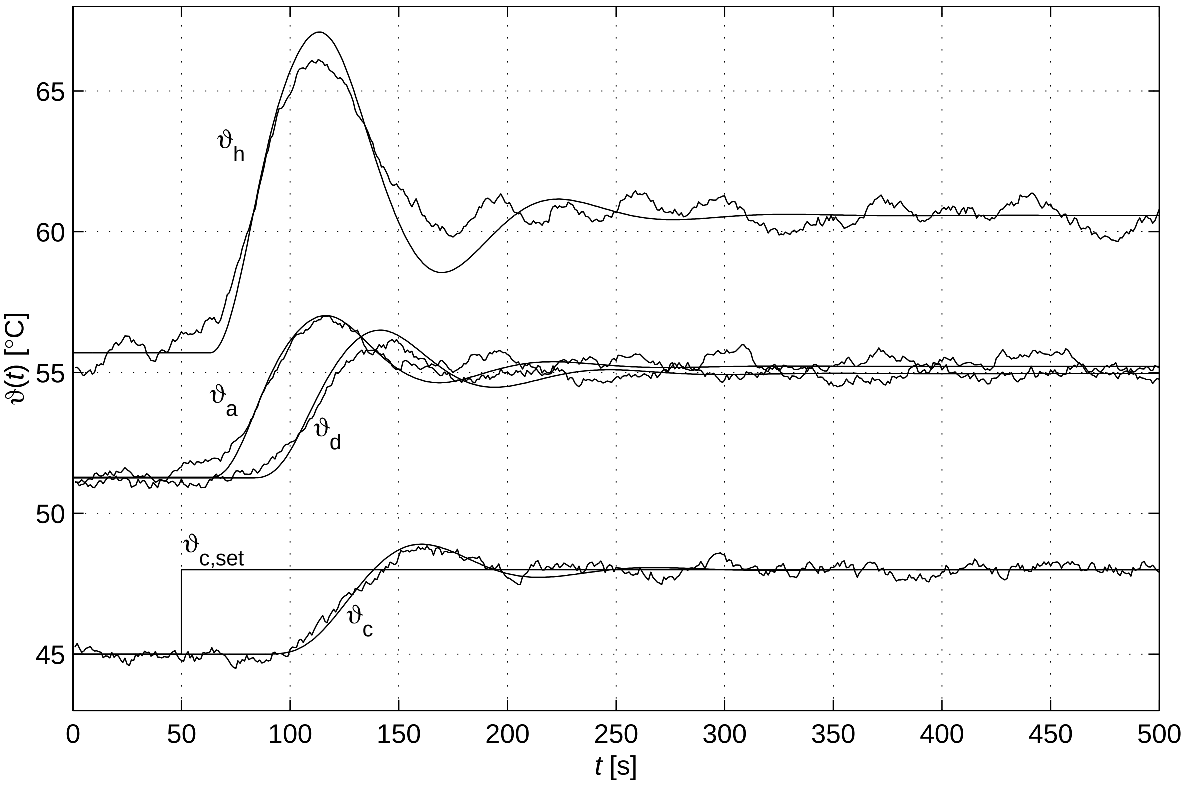

Fig. 12 Comparison of the simulated and the measured set-point responses of the laboratory heating system, feedback coefficients seen in Tab. 2, k = 3, smooth - simulated responses, influanced by noise - measured responses

6. Summary of contributions, conclusions and further directions

This thesis provides several contributions to the analysis and control synthesis of linear time delay systems (TDS). **The first objective** of this thesis, i.e., the design of the **algorithm for computing the roots of the quasipolynomials**, has been solved in chapter 3, where I introduced two algorithms for computing the roots of quasipolynomials. **The first algorithm is based on the extension of Weyl's construction using the argument principle** for computing the number of poles in a particular suspect region of the complex plane (the region in which we suspect some roots being located). **The second algorithm is based on mapping the quasipolynomial in the complex plane**. The algorithm is original, based on constructing the zero-level contours of the real and imaginary parts of the quasipolynomial and locating the intersection points of the contours. On the basis of performed analysis of the features of both the algorithms, I have chosen the latter for the practical realization. The mapping based algorithm has proved to be quite universal. It may be used to compute the roots of polynomials, quasipolynomials and the exponential polynomials. Thus, the **mapping based algorithm can be used to compute poles and zeros of both retarded and neutral systems**. Moreover, also **the essential spectrum of the neutral systems** can be computed. The algorithm can be used to locate the roots in an arbitrarily placed region in the complex plane (not only the rightmost roots can be computed). The most important drawback of the mapping based rootfinder is the incapability to deal with the ill-conditioned (quasi)polynomials. However, this feature is the inherent feature of the non-iterative polynomial rootfinding algorithms. The applicability of the mapping based rootfinder is comparable with the algorithm used in Matlab function *roots*. The incapability of dealing with the ill-conditioned (quasi)polynomials restricts the applicability of the mapping based rootfinder to the low degree (quasi)polynomials (let us say up to n<20). It is due to fact that the higher degree (quasi)polynomials are likely to be ill-conditioned. Most of the algorithms for analysis and control synthesis of TDS presented in this thesis are build on this mapping based rootfinder.

The other objectives of the thesis stated in chapter 2 are solved in chapter 4. First, according to the **objective 2**, the features of the first order anisochronic model are investigated. Especially, the **potentials of approximating the dominant poles using the first order anisochronic model with delay in denominator are studied**. The results achieved show that two parameters of the denominator, i.e., time constant and time delay, allow the dominant couple of the poles to be placed arbitrarily in the complex plane. Taking into account that the system dead time may be approximated by the numerator delay, **the first order anisochronic model may be used to approximate the dynamics of the plants conventionally described by considerably higher order delay-free models**. Secondly, the **first order anisochronic model is further extended to approximate also the effect of the dominant zeros**. The extension is performed by involving an exponential polynomial in the numerator of the transfer function of the anisochronic first order model. By means of the parameters of the exponential polynomial, the dominant zeros may be placed arbitrarily in the complex plane. **Using this model, the dynamics of plants with zero-effect, e. g., non-minimum phase systems can be approximated**. The drawback of involving the exponential polynomial in the numerator of the model is given by the fact that besides the dominant zeros assigned, infinitely many zeros with large imaginary parts (distributed in a vertical strip of the complex plane) are introduced into the system dynamics. On the other hand, as has been shown in the thesis, the model may be used to approximate quite broad class of plant dynamics.

The third objective of the thesis deals with the basic feature of TDS - with the infinite spectrum of poles of TDS. Even though the spectrum of poles of TDS is infinite, the number of poles that determine the dynamics is low as a rule. In the thesis, I have designed an **original pole-significance evaluating criterion**. The criterion **is based on the generalized Heaviside expansion of the input-output transfer function of TDS**. First, the poles of TDS that are closest to the complex plane origin, which are likely to be the dynamics determining poles, are computed using the mapping based algorithm. The significance evaluating criterion evaluate the weighting functions of the transfer functions resulting from the Heaviside expansion. Particularly, **the absolute values of the differences between the maxima and minima of the weighting functions are evaluated**. The larger is the value of the criterion, the pole is more significant. The evaluation of the pole significance is important in the process of selecting the most important modes of the systems dynamics. The selection of the most important modes of the dynamics may be useful in approximating the TDS by a finite order model. It is also useful to define the group of most decisive poles before applying the pole placement method.

In order to fulfil the **fourth objective**, I have investigated features of the methods for pole placement using the proportional feedback from the state variables applied to TDS. First, **the fundamentals of the gradient based state variable feedback design are summarized**. The algorithm presented arises from the linearity of the closed loop characteristic function with respect to the feedback gains and can be used for direct pole placement. The poles being prescribed may be both real and complex conjugate, either single or multiple. The values of the feedback gain coefficients result as the solution of the obtained set of linear equations. The maximum number of poles being prescribed is restricted by the order of the system n, i.e., by the number of available feedback loops. Using the method for direct pole placement, the infinity of the spectrum of TDS has to be taken into consideration. The fact that **we can prescribe the position only to *n* poles, while the other poles (infinitely many) are placed spontaneously**, rather restricts the applicability of the method. To obtain satisfactory result, the following procedure for pole placement using gradient based method is suggested. First, the poles of the original system are computed using, e.g., mapping based rootfinder. Then, the pole significance evaluating criterion is used to define the dynamics determining poles. The third step of the procedure consist in prescribing the new positions to these most significant poles in order to stabilize or improve the system dynamics. In the fourth step of the procedure, applying the computed values of feedback gain coefficients, the new spectrum of the feedback system poles has to be checked using the rootfinder. If some of the non-prescribed poles are placed into the undesirable positions, the result of the procedure cannot be accepted and the whole procedure has to be repeated with the new values of the prescribed poles. If the dominance of the prescribed poles is preserved after the procedure, the poles may be further shifted if the result achieved is not satisfactory yet. In this way, the pole placement is accomplished in several steps. **Even though this pole placement procedure is rather heuristic, it has proved to be efficient in modifying the TDS dynamics**.

Secondly I have modified the method of **gradient based pole placement** so that it might be used in the pole placement method known as **continuous pole placement** originally designed by Michiels, et al.,(2002). The idea of the method consists in shifting only the real parts of the rightmost poles and monitoring the positions of the other poles. Since only the real parts of the poles are prescribed, the characteristic function looses the linearity with respect to the parameters being computed (besides the feedback coefficients, also the imaginary parts of the prescribed poles are computed) if the shifts are prescribed to a complex pole. Therefore, the characteristic function is linearized and the validity of the results achieved are accurate enough only if the prescribed shifts are small. At each step of the continuous pole placement procedure, the rightmost spectrum of the feedback system is computed using the mapping based rootfinder. **The distribution of the feedback system poles resulting from the continuous pole placement procedure is close to the minimal supremum of the real parts of the poles**. Such a result is convenient from the stability point of view. However dynamics determined by this pole distribution have often undesirable features. Since only the real parts of the poles are controlled, the resultant dynamics may be too oscillatory. On the other hand, if all the rightmost poles are real, the resultant dynamics are overdamped. Therefore, in some cases, it is convenient apply the direct pole placement to improve further the feedback system dynamics having resulted from the continuous pole placement. **Both the methods presented in this thesis can be applied to both retarded and neutral systems with both lumped and distributed delays**

In the last chapter, all the **main results of this thesis have been successfully tested on the model of real plant, i.e., laboratory heating system**. To sum up, the main contribution of the thesis is the design of mapping based rootfinder. As has been shown the rootfinder is a powerful tool for computing the spectra of lower order TDS. Particularly, the applicability of the rootfinder to compute the spectra of the neutral systems is a unique result. As has been shown, the knowledge of the spectrum of TDS is quite important (more that in case of delay free systems) in analyzing the system dynamics and designing the control. Note that most of the results presented in this thesis have already bee published. To conclude **all the objectives stated in chapter 2 have been fulfilled**.

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